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Epaminondas E. Panas

Almost Homogeneous Functions:

A Theoretical and Empirical Analysis
with Special Emphasis on Labour Input:
The Case of Swedish Manufacturing
Industries

UPPSALA 1987





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ABSTRACT

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The thesis is a theoretical and empirical study of the almost homogeneous production function. By using the property of almost homogeneity, we are able to derive a class of almost homogeneous production functions. Such a class includes the Cobb-Douglas and the CES type forms as special cases.

An example of this class of functions is discussed with particular attention paid to the specification of labour input. When one attempts to discuss the ease with which different components of labour input may be substituted for one another, an empirical problem arises: the elasticity of substitution between the labour components is constant in the Sato's first stage model.

An extension is suggested. An almost homogeneous function which allows one to empirically estimate the variable elasticity of substitution between labour components is used. This model not only allows one to estimate the elasticity of substitution between two components of the labour input, it also allows for the testing of Sato's first stage.

Results from this investigation are used for the estimation of the two stage production function. This work also presents a comparison of two alternative models concerning the specification of the stochastic error term. Time series estimates of the above models, in connection with the Cobb-Douglas function, are given for nine Swedish manufacturing industries.

The Cobb-Douglas production function restricts the elasticity of substitution between factors of production to unity; a slightly less restricted function is the CES which has been estimated.

This work also attempts to estimate directly the elasticity of substitution, as well as the rates of both labour augmenting and capital augmenting technological progress. It was found that the elasticity of substitution between capital and labour appears to be between zero and one. Also, it was found that the estimates of the rate of labour augmenting are positive while the estimates of capital augmenting appears to have been negative in all nine industries. While Kennedy's hypothesis of induced innovation was useful in explaining the negative rate of capital augmentation, Ahmad's hypothesis explained the bias of technological progress. This type of result is an indication that technological progress was embodied during the 1963-1980 period.

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Uppsala in November 1984

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Introduction and Outline

A familiar approach to production analysis assumes two inputs, say capital (K) and labour (L), which produce an output (y). The general form of the production function which describes the relation between output and inputs can be written as:

$$y = f(K, L). \quad (1.1)$$

Technological progress (t) can be added to this relation, and in the mathematical terms the production function (1.1) can be written as

$$y = f(K, L; t). \quad (1.2)$$

For an empirical determination of the relationship between output and inputs, equation (1.1) needs to be specified further. For example y can be described as a Cobb-Douglas production function

$$y = AK^\alpha L^\beta \quad (1.3)$$

with $0 < \alpha < 1$ and $0 < \beta < 1$ or as a CES production function

$$y = \gamma [\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho} \quad (1.4)$$

where γ , δ and ρ denote the efficiency, intensity and substitution parameters, respectively.

A number of theoretical and empirical papers on production functions have been written since Cobb-Douglas published "A Theory of Production". Uzawa¹ and Sato² have both elaborated on this original work. Uzawa proposed a production function that is a Cobb-Douglas type of function which in turn incorporates CES functions in their components, i.e.,

$$y = AK^{a_1} L^{a_2} \quad (1.5)$$

¹ See Uzawa (1962).

² See Sato (1967). See also Bergström (1982).

where

$$K = [\gamma(KM)^{-\rho} + (1-\gamma)(KB)^{-\rho}]^{-1/\rho} \quad (1.6)$$

and KM (machinery), KB (buildings) are the components of K .

Sato extended the Uzawa function considering output as a CES function of CES functions:

$$y = A[\delta L^{-\mu} + (1-\delta)K^{-\mu}]^{-1/\mu} \quad (1.7)$$

where K is given by (1.6).

These methods, in addition to earlier production analysis, are similar in the following respect: they all assume a homogeneous labour input. Homogeneity of labour input arises if we can directly add the components of the labour input, or equivalently, if the elasticity of substitution³ between two labour components is infinite.

In most empirical studies of the production function, the aggregate inputs of capital (K) and labour (L) are treated as homogeneous factors of production.

In practice, however, neither capital nor labour is homogeneous. This lack of homogeneity raises a problem: the different components constituting the inputs in question are not simply additive.

Yet it remains true that apart from a few exceptions, the main body of empirical production functions are estimates in a way which avoids the problems of aggregation of the labour input.⁴

Until quite recently, most econometric studies of production had not emphasized the accurate specification of the labour input. As we have noted, the principal role assigned to the production function was the

³ The elasticity of substitution between two components of labour, L_i and L_j , denoted by σ , is defined as

$$\sigma = \frac{d \log (L_i / L_j)}{d \log (W_i / W_j)}.$$

In other words, σ represents the percentage change in the ratio of labour quantities, $d(L_i/L_j)/L_i/L_j$, per one percent change in the labour price ratios,

$$\frac{d(W_i/W_j)}{W_i/W_j}.$$

⁴ Solow (1960) has been concerned with the measurement question when discussing a Cobb-Douglas production function: "There are index number problem involved in the measurement of each of the variables".

In addition, Berndt and Christensen (1974) suggested that "the aggregation of diverse capital inputs has drawn more criticism, most notably from the Cambridge school, than the aggregation of diverse labour inputs".

aggregation of the capital input. The labour input was assumed homogeneous. Recent work has indicated, however, that the specification of the labour input in fact is more complex.

Walters⁵ observes that production function studies have overlooked differences in labour inputs because of data limitations and that this practice introduces a serious bias in the measurement of labour input. Walters suggests that the problem is “to aggregate different kinds of labour according to skill, sex and age, and, second, to allow for changing weights over time”.

For example, the labour input can be disaggregated into two components: WE (wage-earners) and SE (salaried employees). The production function (1.2) then becomes

$$y = f(K, WE, SE; t). \quad (1.8)$$

This specification implies that the labour input is not a homogeneous factor of production. Equation (1.8) establishes a link between the occupational structure of the labour input and output. The specification (1.8) includes (1.2) as a special case in the sense that the two labour components, WE and SE, are aggregated assuming an elasticity of substitution between them of infinity, i.e., a linear function $L = WE + SE$ so that the factors are perfectly substitutable.

In general terms, most empirical production studies can be divided into two broad categories with respect to the labour input. In the first type, the labour input is estimated as a simple sum of occupational components. Variations of this technique have been used by Griliches-Ringstad⁶ and by Denison.⁷ For example, according to Denison's approach, aggregate labour is a weighted sum of labour of different occupational components, the weights being the relative prices.

An aggregate labour input which satisfies these requirements is:

$$L = L_1 + W_2 L_2 + \dots + W_n L_n \quad (1.9)$$

where

L_i = the labour components;

W_i = weights; the ratios of earning of the labour component i to the earnings of the labour component 1.

The second category of empirical research emphasizes the importance of

⁵ See Walters (1963).

⁶ See Griliches and Ringstad (1971).

⁷ See Denison (1962).

occupational components. In these studies, such as Bowles,⁸ Psacharopoulos-Hinchliffe⁹ and Dougherty,¹⁰ an attempt was made to estimate the labour input as a CES function of different labour components.

Bowles was concerned with the hypothesis of an infinity elasticity of substitution between different labour components. The aggregate function for labour was assumed to be a CES function of the individual components of labour and he estimated the elasticity of substitution from the equation

$$\log\left(\frac{W_i}{W_j}\right) = a + b \log\left(\frac{L_i}{L_j}\right) + u \quad (1.10)$$

where

W_i = average wage rate of the i th labour component

L_i = the i th labour component

u = stochastic error term.

The elasticity of substitution between the two components of labour, σ , is given by

$$\sigma = -1/b. \quad (1.11)$$

On these grounds, Bowles observes that:

... The evidence presented lends support to the view that elasticities of substitution among different labor inputs are high, and that for many purposes there may be no significant empirical loss in accepting as a working assumption the proposition that the elasticities of substitution among labour inputs are infinite.

This implies, according to Bowles, a simple method of aggregation, i.e., the two labour components can be linearly aggregated to form the simple measure of labour input: $L = WE + SE$.

Dougherty used Bowles' technique for the purpose of constructing more detailed labour aggregation functions. In the case of Dougherty's study, the labour function was assumed to be a CES function whose components could either be the individual components or be themselves CES functions of the components.

The parameters of the equation $L = (a_1 L_1^\theta + a_2 L_2^\theta)^{1/\theta}$ were estimated from

$$\log\left(\frac{W_1}{W_2}\right) = a + (\theta - 1) \log\left(\frac{L_1}{L_2}\right) + u \quad (1.12)$$

and $-\frac{1}{\theta - 1}$ is a maximum-likelihood estimate of σ .

⁸ See Bowles (1970).

⁹ See Psacharopoulos and Hinchliffe (1971).

¹⁰ See Dougherty (1972).

He concluded that:

... the estimates of the elasticity of substitution are significantly different from infinity at the 5 percent level ...

Hence, the linear aggregation form ($L = L_1 + L_2$) would be inappropriate for aggregating the labour components.

The main problem of these studies (Bowles and Dougherty) is the neglect of the role of the variable elasticity of substitution between labour components. The common theme of these papers is that they both stress the importance of heterogeneity of the labour input. But they neglect the very important question of the elasticity of substitution between capital and labour. That is, there is no link between the findings of labour aggregation and the production function.

The statistical findings of these studies reflect situations where the labour input is far from being homogeneous. Dougherty's study clearly establishes that "more sophisticated functions could have been considered".

As a result it is not difficult to conclude that further research is necessary. If it is possible to develop a model to better explain why the heterogeneity occurs, then such a framework can be used in the production analysis. That is the objective of this study.

With this background, we can now turn to a more specific discussion of the model that is developed in the following chapters.

The formal structure of the production model developed in this study consists of two stages. The first explains the aggregate labour input, while the second estimates production functions where the labour input was derived from the first stage.

The CES functional form has been chosen by the above researchers in preference to the Cobb-Douglas form. But the CES form does impose its own restrictions, in particular that the elasticity of substitution between any two labour components is the same.

The purpose of this study is to develop a simple model that composes the different labour components into an aggregate labour input. In general terms, our procedure can be outlined very briefly. From the theory of almost homogeneous functions one can derive an expression for the labour input which incorporates only two components: wage earners (= WE) and salaried employees (= SE).

The almost homogeneous functions, which will be utilized below, is expressed by the following relation:

$$L = \delta(WE)^{1/k_1} + (1-\delta)(SE)^{1/k_2}. \quad (1.13)$$

What is the implication of this type (1.13) of aggregation on the elasticity of substitution between WE and SE?

It can be shown that the elasticity of substitution between WE and SE is a variable elasticity of substitution (V.E.S.). In technical terms, variable elasticity of substitution is equivalent to the existence of a more general function than a CES.

All previous empirical studies of aggregate labour input have been limited to a CES functional form, that is, aggregate labour input is a CES function of the labour components.

It is useful to point out how the almost homogeneous labour model differs from other models. Two points were highlighted in the review of the Bowles and Dougherty models. First, it was noted that the elasticity of substitution between two labour components may not be constant. That is, only the V.E.S. type of function has the possibility to overcome this problem. Second, in the more complex case with several labour components, the almost homogeneous function has the property that each pair of components has a distinct partial elasticity of substitution. This is much more realistic than assuming that the partial elasticity of substitution between, say, managers and technical workers is the same as between technical workers and manual workers, as would be the case with CES functions. It should be clear that these differences are crucial to the aggregation problem.

In fact, a CES measure of the labour input cannot be obtained. Fortunately, however, the almost homogeneous function (1.13) developed here can assist in the aggregation procedure. The measure of the labour input is based on empirical tests which the theory of almost homogeneous function suggests. The data for these tests, as well as for other estimates discussed in this study, are based on yearly observations from 1963–1980 for the Swedish manufacturing industries.

Two important conclusions are drawn from the empirical tests of almost homogeneous labour functions. First, the existence of heterogeneity of labour input as an empirically important phenomenon is confirmed. Second, the validity of the almost homogeneous function that forms the basis of the empirical tests is also supported.

Therefore, the method of aggregating labour components developed here can be viewed as a logical extension of previous studies in this area.

This study is organized as follows:

Chapter 2 contains a discussion of almost homogeneity and how it relates to production analysis. Also through the property of almost homogeneity, a class of almost homogeneous production functions is derived. The mathematical property of almost homogeneity, introduced by Lau,¹¹ will thus be used for the first time to derive a class of production functions.

¹¹ See Lau (1972, 1978).

In addition, the research reported in Chapter 2 has addressed itself to the following questions:

- (i) What is the geometrical meaning of almost homogeneous functions?
- (ii) How can almost homogeneous functions be compared to the more traditional production functions such as Cobb-Douglas and CES with respect to the elasticity of substitution, scale elasticity, and homotheticity?
- (iii) How can the various assumptions concerning the scale elasticity be applied to almost homogeneous production functions? Or under what conditions do the almost homogeneous production functions satisfy the Zellner-Revankar differential equation?

Chapter 3 develops the methodology for estimating the almost homogeneous labour input function (1.13), and discusses the statistical criteria for choosing this function.

This study is the first to examine the WE-SE relationship within the class of almost homogeneous functions.

Two main aims are described in Chapter 3: the economic problems of a Cobb-Douglas production function are surveyed and analyzed and an almost homogeneous labour function is estimated and tested. Then the appropriate labour input is introduced into the Cobb-Douglas production function.¹² This production function then is estimated multiplicatively and additively. When there are no a priori reasons for a choice between the two alternatives stochastic error terms, the specification can be determined empirically. Thus, attention is paid to the specification of the stochastic error term.

It should be noted here that the empirical results of Chapter 3 are based on a Cobb-Douglas production function, in other words that the elasticity of substitution between capital and labour is equal to one.

In the next chapter, Chapter 4, production functions which do not necessarily make such an assumption regarding elasticities will be examined.

Chapter 4 deals empirically with the question of elasticities of substitution between capital and labour. In Chapter 4 further aspects of elasticity of substitution and technological progress are considered. The objectives of Chapter 4 can thus be summarized as: (a) to estimate the elasticity of substitution between capital and labour; (b) to estimate the bias of technological progress; (c) to estimate directly the rates of factor augmentation as well as the elasticity of substitution (The Kalt-Bergström-Melander model)¹³ by using the "Seemingly Unrelated Regressions Equations" (SURE)

¹² This is an extension of Uzawa's model, when labour is specified as an almost homogeneous function.

¹³ See Kalt (1978). See also Bergström (1982) and Bergström-Melander (1982).

technique and (d) to extend the above model by estimating a dynamic generalization of the Kalt–Bergström–Melander model. The dynamisation used here generalizes the SURE model.

Finally, the conclusions are presented in Chapter 5.

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On the General Class of Almost Homogeneous Production Functions

2.1 Introduction

Lau (1972, 1978) initially introduced the concept of almost homogeneous functions in economics in order to characterize the structure of profit functions. In this chapter we shall discuss the definition and the properties of almost homogeneous functions.

Also in this chapter it will be shown how production functions can be derived from economic relationship. The crucial point is that it is not necessary to specify the production function a priori but rather let it evolve from the property of almost homogeneity. Expressing the property of almost homogeneity as a partial differential equation, a class of production functions is obtained by integrating this partial differential equation.

In section 2.2 we will outline the definition of an almost homogeneous (AH) function. In section 2.3, using the definition of almost homogeneity, the general functional form of the almost homogeneous production function is derived. In section 2.4 we shall discuss the neoclassical properties of almost homogeneous production function in a two factor context. In this case the functional form was derived from the definition of almost homogeneity and is the same as the Dhrymes-Kurz (1964) production function (which is an almost homogeneous function). Similarly, the Mukerji (1963) production function can be derived from the definition of almost homogeneity. The Dhrymes-Kurz and Mukerji production function have been analyzed by Hanoch (1971, 1975, 1978). The properties of homotheticity and the relationship of homotheticity to almost homogeneity are discussed in section 2.5. Also, in section 2.6 we show, using the concept of pseudo homotheticity and given an almost homogeneous production function, that it is possible to define a transformation which satisfy the well known Zellner-Revankar (1969) differential equation.

Finally, the method of deriving a non-almost homogeneous production function is discussed in section 2.7 with the aid of Box-Cox (1962) transformations.

2.2 Almost homogeneous functions

In this section the definition of almost homogeneity will be examined in detail, in order to illustrating the potential of this property.

It is well known that a function $f(x_1, \dots, x_m)$ is homogeneous of degree n if and only if equation (2.1) holds:

$$f(\lambda x_1, \dots, \lambda x_m) \equiv \lambda^n f(x_1, \dots, x_m). \quad (2.1)$$

In the case of homogeneous functions returns to scale are characterized by the parameter n . If $n=1$, then the function f is linear homogeneous.

But since f has been assumed to be a homogeneous function of degree n , an application of Euler's theorem to (2.1) yields:

$$x_1 \frac{\partial f}{\partial x_1} + \dots + x_m \frac{\partial f}{\partial x_m} \equiv n f. \quad (2.2)$$

Definition: A function $f(x_1, \dots, x_m)$ is almost homogeneous (AH) or pseudo-homogeneous¹ if equation (2.3) holds:

$$f(\lambda^{k_1} x_1, \dots, \lambda^{k_m} x_m) \equiv \lambda^k f(x_1, \dots, x_m). \quad (2.3)$$

for every $\lambda > 0$ and where $k_1, \dots, k_m$ are constants.

The application of almost homogeneous functions in economics was first suggested by Lau.² Lau also shows that Euler's theorem applies in the following modified form:

$$k_1 x_1 \frac{\partial f}{\partial x_1} + \dots + k_m x_m \frac{\partial f}{\partial x_m} \equiv k f(x_1, \dots, x_m). \quad (2.4)$$

Now, let $y = f(x_1, x_2)$ be an AH production function in two input spaces, i.e.

$$\lambda^k \cdot y = f(\lambda^{k_1} x_1, \lambda^{k_2} x_2). \quad (2.5)$$

We have restricted ourselves to the case of two inputs in order to ease interpretation. However, extensions to more than two inputs are straightforward.

In the case of (2.5) the modified Euler's theorem (2.4) can be written as

$$k_1 x_1 \frac{\partial f}{\partial x_1} + k_2 x_2 \frac{\partial f}{\partial x_2} = k f(x_1, x_2). \quad (2.6)$$

if k_1 , k_2 and k are fixed, the differentiation of both sides of the modified Euler's equation with respect to x_1 and x_2 yields

¹ See Aczel (1966), Kolm (1976), Lau (1972, 1978) and Sato (1977).

² See Aczel (1966), Kolm (1976), Lau (1972, 1978) and Sato (1977).

$$k_1 \frac{\partial f}{\partial x_1} + k_1 x_1 \frac{\partial^2 f}{\partial x_1^2} + k_2 x_2 \frac{\partial^2 f}{\partial x_1 \partial x_2} = k \frac{\partial f}{\partial x_1}$$

or

$$k_1 x_1 \frac{\partial^2 f}{\partial x_1^2} + k_2 x_2 \frac{\partial^2 f}{\partial x_1 \partial x_2} = (k - k_1) \frac{\partial f}{\partial x_1}. \quad (2.7)$$

Similarly,

$$k_1 x_1 \frac{\partial^2 f}{\partial x_1 \partial x_2} + k_2 x_2 \frac{\partial^2 f}{\partial x_2^2} = (k - k_2) \frac{\partial f}{\partial x_2}. \quad (2.8)$$

From (2.7) and (2.8) it follows that $\partial f / \partial x_1$ and $\partial f / \partial x_2$ are almost homogeneous functions of degrees

$$k_1, k_2, k - k_1$$

and

$$k_1, k_2, k - k_2$$

respectively.

In the following we limit ourselves to a further analysis of the almost homogeneous definition. The almost homogeneous function which we are going to study geometrically in this section is expressed by the relation (2.5).

If $k_1 = k_2 = k_0$, then

$$\lambda^k y = f(\lambda^{k_0} x_1, \lambda^{k_0} x_2)$$

and for $\lambda^{k_0} = 1/x_1$ we have

$$x_1^{k/k_0} \cdot y = f\left(1, \frac{x_2}{x_1}\right)$$

or

$$y = x_1^{k/k_0} f\left(1, \frac{x_2}{x_1}\right).$$

This relationship implies that f is homogeneous of degree k/k_0 .

If also $k_1 = k_2 = k$, then f is homogeneous of degree 1. This means that homogeneous functions are a special case of almost homogeneous func-

tions. Therefore, homogeneous functions are a subset of a general class of almost homogeneous function.

In order to clarify the economic implications of almost homogeneity consider that x^0 is a given initial factor proportion:

$$x^0 = \frac{x_2^0}{x_1^0}.$$

If we transform both inputs to $(\lambda^{k_1} x_1^0, \lambda^{k_2} x_2^0)$, then new proportion will be

$$x^* = \frac{\lambda^{k_2} x_2^0}{\lambda^{k_1} x_1^0} = \lambda^{k_2 - k_1} x^0 \quad (2.9)$$

and this will be different from the previous expression if $k_2 - k_1 \neq 0$. If $k_2 - k_1 > 0$, then the factor proportion, x^* , increases from zero to ∞ .

Let us more thoroughly examine the proportional variation. Given the initial inputs x_1^0, x_2^0 , then there exists a real non-negative number θ such that

$$\theta = \frac{x_2^{0^{1/k_2}}}{x_1^{0^{1/k_1}}}$$

or

$$x_2^{0^{1/k_2}} = \theta x_1^{0^{1/k_1}}. \quad (2.10)$$

Now, if we transform both initial inputs to

$$x_1 = \lambda^{k_1} x_1^0, x_2 = \lambda^{k_2} x_2^0$$

then the question arises: did the transformed inputs x_1 and x_2 satisfy similar relationships (2.10) as x_1^0 and x_2^0 ?

The answer will be yes, for

$$x_2^{1/k_2} = \lambda x_2^{0^{1/k_2}}$$

and since $x_2^{0^{1/k_2}} = \theta x_1^{0^{1/k_1}}$ we have

$$x_2^{1/k_2} = \lambda x_2^{0^{1/k_2}} = \lambda \theta x_1^{0^{1/k_1}} = \theta x_1^{1/k_1}$$

that is:

$$x_2^{1/k_2} = \theta x_1^{1/k_1}. \quad (2.11)$$

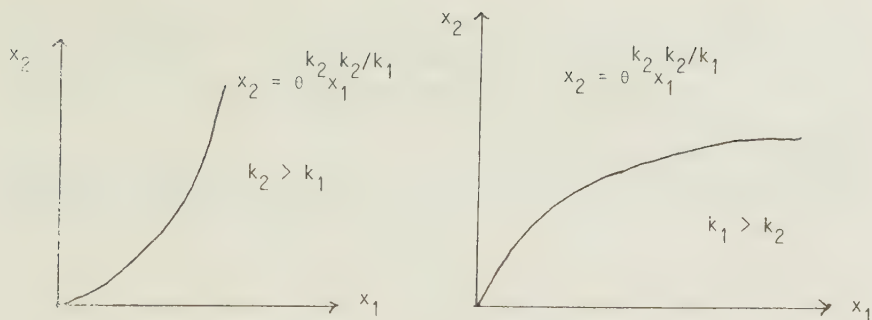


Fig. 1

Thus, as we change both inputs by λ^{k_1} and λ^{k_2} so as to change output by λ^k , then x_2^{1/k_2} and x_1^{1/k_1} always appear in the relation (2.11) in which x_2^0 and x_1^0 exist. Therefore, in order to increase output continuously by $k\%$, we can do it by increasing x_1 by $k_1\%$ and x_2 by $k_2\%$, i.e. by appropriately changing the factor proportions.

The implication is that we can define the almost homogeneous production function as follows: a production function $y = f(x_1, x_2)$ is said to be almost homogeneous if output rises by $k\%$ whenever x_1 and x_2 increase by $k_1\%$ and $k_2\%$ respectively.

Note that in the case of AH production functions, when the inputs increase by the same proportion ($k_1 = k_2 = k_0$) the output generally does not increase by the same proportion.

The corresponding curves are shown in Fig. 1 for $k_2 > k_1$ and $k_1 > k_2$.

2.3 The analytical form of almost homogeneous production functions

It is well known that from the definition of the elasticity of substitution the functional form of the production function can be derived. It is less well known that by employing the definition of other characteristics of production (for example scale elasticity or almost homogeneity) a specific form of production function can be derived. In this section a simple method is described, based on a partial differential equation—the modified Euler equation according to Lau—to specify a production function characterized by almost homogeneity. This has not been done in the past.

A special case of the solution to the modified Euler equation which has been applied by Dhrymes and Kurz³ and Hanoch³ also points out several theoretical properties of this function.

³ See Dhrymes and Kurz (1964), Hanoch (1971, 1975, 1978).

The modified Euler equation (2.6) is a first order quasi-linear partial differential equation involving x_1 , x_2 and f .

To solve the quasi-linear partial differential equation we formulated the Lagrange equation⁴.

$$\frac{dx_1}{k_1 x_1} = \frac{dx_2}{k_2 x_2} = \frac{df}{kf}. \quad (2.12)$$

From (2.12) we have

$$\frac{dx_1}{k_1 x_1} = \frac{dx_2}{k_2 x_2} \Leftrightarrow \frac{1}{k_1} \log x_1 = \frac{1}{k_2} \log x_2 + \log c_1 \Leftrightarrow \log \frac{x_1^{1/k_1}}{x_2^{1/k_2}} = \log c_1 \Leftrightarrow \frac{x_1^{1/k_1}}{x_2^{1/k_2}} = c_1 \quad (2.13)$$

and

$$\frac{dx_2}{k_2 x_2} = \frac{df}{kf} \Leftrightarrow \frac{f^{1/k}}{x_2^{1/k_2}} = c_2. \quad (2.14)$$

The general solution of the modified Euler's theorem has the form

$$\varphi(c_1, c_2) = 0$$

or

$$\varphi\left(\frac{x_1^{1/k_1}}{x_2^{1/k_2}}, \frac{f^{1/k}}{x_2^{1/k_2}}\right) = 0$$

or

$$\frac{f^{1/k}}{x_2^{1/k_2}} = \varphi^*\left(\frac{x_1^{1/k_1}}{x_2^{1/k_2}}\right). \quad (2.15)$$

The inverse problem can now be proved, i.e. if $\varphi(u, v) = 0$ where $u = x_1^{1/k_1}/x_2^{1/k_2}$ and $v = f^{1/k}/x_2^{1/k_2}$ find the partial differential equation which corresponds to $\varphi(u, v) = 0$.

Differentiation of $\varphi(u, v) = 0$ gives us:

$$\frac{\partial \varphi}{\partial u} \left\{ \frac{1}{k_1} \frac{x_1^{1/k_1-1}}{x_2^{1/k_2}} \right\} + \frac{\partial \varphi}{\partial v} \left\{ \frac{1}{k} \frac{f^{1/k-1}}{x_2^{1/k_2}} \cdot \frac{\partial f}{\partial x_1} \right\} = 0$$

⁴ See Lax (1951) and Smirnov (1964).

$$\frac{\partial \varphi}{\partial u} \left\{ -\frac{1}{k_2} \frac{x_1^{1/k_1} x_2^{1/k_2-1}}{x_2^{2/k_2}} \right\} + \frac{\partial \varphi}{\partial v} \left\{ -\frac{1}{k_2} \frac{f^{1/k} x_2^{1/k_2-1}}{x_2^{2/k_2}} + \frac{1}{k} \frac{f^{1/k-1}}{x_2^{1/k_2}} \frac{\partial f}{\partial x_2} \right\} = 0. \quad (2.15.1)$$

If we consider (2.15.1–2.15.2) as two simultaneous systems of equation in $\partial \varphi / \partial u$, $\partial \varphi / \partial v$ then the necessary and sufficient condition in order for the system (2.15.1, 2.15.2) to have a non trivial solution is:

$$\begin{vmatrix} \frac{1}{k_1} \frac{x_1^{1/k_1-1}}{x_2^{1/k_2}} & \frac{1}{k} \cdot \frac{f^{1/k-1}}{x_2^{1/k_2}} \cdot \frac{\partial f}{\partial x_1} \\ -\frac{1}{k} \cdot \frac{x_1^{1/k_1}}{x_2^{1/k_2+1}} & -\frac{1}{k_2} \frac{f^{1/k}}{x_2^{1/k_2+1}} + \frac{1}{k} \cdot \frac{f^{1/k-1}}{x_2^{1/k_2}} \cdot \frac{\partial f}{\partial x_2} \end{vmatrix} = 0$$

or

$$k_1 x_1 \frac{\partial f}{\partial x_1} + k_2 x_2 \frac{\partial f}{\partial x_2} = k f.$$

Thus we have found the modified Euler's theorem.

One special case of (2.15) is obviously the following

$$\frac{f^{1/k}}{x_2^{1/k_2}} = \alpha \frac{x_1^{1/k_1}}{x_2^{1/k_2}} + \beta$$

from which it follows that

$$f = (\alpha x_1^{1/k_1} + \beta x_2^{1/k_2})^k. \quad (2.16)$$

Multiplying x_1 by λ^{k_1} and x_2 by λ^{k_2} it is seen that (2.16) is indeed an almost homogeneous function.

2.4 The almost homogeneous production function:

$$y = (\alpha x_1^{1/k_1} + \beta x_2^{1/k_2})^k$$

As a further investigation of the almost homogeneous production function (2.16) it is useful to examine under what conditions it satisfies the neoclassical properties. It is now necessary to give briefly these properties.

Consider the production function

$$y = f(K, L).$$

Production is zero if the capital or labour is zero:

$$f(0, L) = 0, f(K, 0) = 0.$$

Marginal productivities are positive. Symbolically

$$\frac{\partial f}{\partial K} = f_K > 0; \quad \frac{\partial f}{\partial L} = f_L > 0.$$

Within the economic region each marginal product should decrease as each input increases. This is a sufficient condition for equilibrium which implies that the isoquants are convex to the origin.

The results of this section can be summarized as: the production function (2.16) has the following theoretical properties:

- positive marginal products;
- downward sloping marginal product curves over the relevant ranges of the inputs;
- almost homogeneity;
- variable elasticity of substitution.

Moreover, it includes the CES function as a special case.

2.4.1 Marginal products

The marginal products of capital (x_1) and labour (x_2) can be obtained from (2.16):

$$\frac{\partial f}{\partial x_1} = f_1 = \frac{k}{k_1} \alpha (\alpha x_1^{1/k_1} + \beta x_2^{1/k_2})^{k-1} x_1^{(1-k_1)/k_1} \quad (2.17)$$

$$\frac{\partial f}{\partial x_2} = f_2 = \frac{k}{k_2} \beta (\alpha x_1^{1/k_1} + \beta x_2^{1/k_2})^{k-1} x_2^{(1-k_2)/k_2}. \quad (2.18)$$

It is obvious from (2.17) and (2.18) that

$$f_1 > 0 \text{ if } \frac{k}{k_1} > 0 \quad (2.19)$$

$$f_2 > 0 \text{ if } \frac{k}{k_2} > 0. \quad (2.20)$$

If the k 's are all negative or positive then the marginal products are positive.

Mathematically, convexity of the isoquants can be examined by considering the curve $x_2 = x_2(x_1)$. The negative slope of this curve is indicated by

$$\frac{dx_2}{dx_1} < 0$$

and convexity by

$$\frac{d^2x_2}{dx_1^2} > 0.$$

The slope of an isoquant in the x_1, x_2 space is

$$\frac{dx_2}{dx_1} = \frac{f_1}{f_2}$$

$$\left. \frac{d^2x_2}{dx_1^2} \right|_{f=\text{const.}} = \frac{1}{f_2^3} [-f_2^2 f_{11} + 2f_1 f_2 f_{12} - f_1^2 f_{22}]. \quad (2.21)$$

Consider again the almost homogeneous function (2.16) then (2.21) can be written as

$$\frac{d^2x_2}{dx_1^2} = \frac{\alpha k_2}{\beta k_1^2} \cdot \frac{x_1^{(1-2k_1)/k_1}}{x_2^{(1-k_2)/k_2}} \left[(1-k_1) + \frac{\alpha}{\beta} (1-k_2) x_1^{1/k_1} x_2^{-1/k_2} \right]. \quad (2.22)$$

If the k 's are negative, it follows from the above relation that we have convex isoquants for every $k_i < 0$.

If the k 's are positive, the terms inside the bracket in (2.22) must be negative, i.e.,

$$1-k_1 < 0 \text{ and } 1-k_2 < 0 \quad (2.23)$$

or

$$k_1 > 1 \text{ and } k_2 > 1. \quad (2.24)$$

In this case (2.24) we have a sufficient condition. If $k_1 = k_2 = 1$ then we have straight-line isoquants.

2.4.2 The meaning of α and β

To understand the meaning of the parameters α and β , consider the implications for output if they are both doubled. This gives:

$$y = (2\alpha x_1^{1/k_1} + 2\beta x_2^{1/k_2})^k = 2^k (\alpha x_1^{1/k_1} + \beta x_2^{1/k_2})^k \quad (2.25)$$

and it is clear that output will increase when k is positive but will decrease when k is negative.

It can be shown that the parameters k_1 , k_2 and k are independent of the units of measurement of the variables, however, the values of α and β depend on the units of the relevant variables.

Instead of (2.16) from above, i.e.

$$y = (\alpha x_1^{1/k_1} + \beta x_2^{1/k_2})^k \quad (2.16)$$

the following model is used:

$$y^* = (\alpha^* x_1^{*1/k_1} + \beta^* x_2^{*1/k_2})^k$$

with

$$y^* = \frac{y}{\bar{y}}, x_1^* = \frac{x_1}{\bar{x}_1}, x_2^* = \frac{x_2}{\bar{x}_2} \quad (2.26)$$

where $\bar{y}$, $\bar{x}_1$, and $\bar{x}_2$ denote the arithmetic means of the variables y , x_1 , and x_2 respectively.

From (2.16) and (2.26) it follows that:

$$y^* \bar{y} = (\alpha \bar{x}_1^{1/k_1} x_1^{*1/k_1} + \beta \bar{x}_2^{1/k_2} x_2^{*1/k_2})^k \Leftrightarrow$$

$$y^* = \left(\frac{\alpha \bar{x}_1^{1/k_1}}{\bar{y}^{1/k}} x_1^{*1/k_1} + \frac{\beta \bar{x}_2^{1/k_2}}{\bar{y}^{1/k}} x_2^{*1/k_2} \right)^k \Leftrightarrow$$

$$\frac{\alpha \bar{x}_1^{1/k_1}}{\bar{y}^{1/k}} = \alpha^*. \quad (2.27.1)$$

$$\frac{\beta \bar{x}_2^{1/k_2}}{\bar{y}^{1/k}} = \beta^*. \quad (2.27.2)$$

Dividing (2.27.1) and (2.27.2) we find

$$\frac{\alpha}{\beta} \cdot \frac{\bar{x}_1^{1/k_1}}{\bar{x}_2^{1/k_2}} = \frac{\alpha^*}{\beta^*}$$

which shows that the values of α and β change with any change in the units of the relevant variables. Consequently, the parameters α and β depend on the units in which inputs are measured, however, the parameters k_1 , k_2 and k are independent of the units of measurement of the variables.

2.4.3 The scale elasticity

For a two-factor production function, $y=f(x_1, x_2)$, the scale elasticity of output, ε , is defined by:

$$\varepsilon = \frac{\partial f}{\partial x_1} \cdot \frac{x_1}{y} + \frac{\partial f}{\partial x_2} \cdot \frac{x_2}{y}. \quad (2.28)$$

From (2.16) and (2.28) it follows that:

$$\varepsilon = \frac{x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2}}{f(x_1, x_2)} = k \cdot \frac{\frac{\alpha}{k_1} x_1^{1/k_1} + \frac{\beta}{k_2} x_2^{1/k_2}}{\alpha x_1^{1/k_1} + \beta x_2^{1/k_2}}. \quad (2.29)$$

What conditions must be fulfilled for scale elasticity to remain constant?

If we impose the following restriction

$$x_2^{1/k_2} = \theta x_1^{1/k_1} \quad (2.30)$$

then, the scale function can be written

$$\varepsilon = \text{constant} = \frac{k}{k_1 k_2} \cdot \frac{\alpha k_2 + \beta k_1 \theta}{\alpha + \beta \theta}. \quad (2.31)$$

From (2.31) it follows that the scale elasticity is constant along the parabolic curves (2.30).

Furthermore, it is of interest to examine if the production function (2.16) satisfies the ultra-passum law.⁵

In the long-run, the firm chooses the input combination that minimizes the cost of producing a given output. The necessary conditions for minimizing variable costs

$$C = p_1 x_1 + p_2 x_2$$

are

$$p_i = \mu \frac{\partial f}{\partial x_i}, i = 1, 2 \quad (2.32)$$

where C is total cost, p_1, p_2 are the prices of two inputs and μ is the marginal cost.

These conditions are also sufficient if $y=f(x_1, x_2)$ is the neoclassical production function.

Equation (2.32) defines a curve in the input space which is called the *expansion path*, i.e. the points (x_1, x_2) satisfying (2.32) constitute a path.

Furthermore, a point (x_1, x_2) in the input space corresponds to a firm's long-run equilibrium if the minimum of average cost occurs when $\varepsilon(x_1, x_2)=1$.

Points (x_1, x_2) in the input space which satisfy the condition $\varepsilon(x_1, x_2)=1$

⁵ See Frisch (1965).

are *technically optimal* and the geometric locus of such points is defined as the *technically optimal scale*⁶ or “Wicksell-line”.⁶

In Førsund it is shown that the curve of technically optimal scale is identical with the concepts “Wicksell-line” and “Technique-line” introduced by Bentzel and Johansson.⁷

Frisch⁸ used in his production analysis the “Regular Ultra Passum Law”. This crucial assumption says that the equation $\varepsilon(x_1, x_2)=1$ defines an $n-1$ dimensional surface which is downward sloping, so that $\varepsilon(x_1, x_2)>1$ below it and $\varepsilon(x_1, x_2)<1$ above it, along any upward sloping path of inputs. The ultra-passum law requires that:

$$\frac{\partial \varepsilon}{\partial x_i} < 0, \quad i = 1, 2;$$

$$\left. \frac{\partial x_1}{\partial x_2} \right|_{\varepsilon=1} < 0.$$

We now examine if the almost homogeneous production function (2.16) satisfies the ultra-passum law.

Let us assume that the k 's are negative (or positive). The expansion paths corresponding to this equation are given by the equation:

$$x_2 = \left(\frac{\alpha k_2 p_2}{\beta k_1 p_1} \right)^{k_2/(1-k_2)} \cdot x_1^{\frac{k_2(1-k_1)}{k_1(1-k_2)}} \quad (2.33)$$

and the Wicksell curve is

$$x_2 = \left[\frac{\alpha k_2 (k_1 - k)}{\beta k_1 (k - k_2)} \right]^{k_2} x_1^{k_2/k_1}. \quad (2.34)$$

We see that the (AH) production function (2.16) does not satisfy the “Regular Ultra-passum Law”, since the Wicksell curve is positively sloped, i.e.

$$\left. \frac{\partial x_2}{\partial x_1} \right|_{\varepsilon=1} > 0.$$

In conclusion, as Hanoch⁹ explains the “Regular Ultra-passum Law” is neither necessary nor sufficient for $f(x_1, x_2)$ to be classical (a production function which yields U-shaped average cost curves).

⁶ See Førsund (1971, 1974).

⁷ See Bentzel and Johanson (1959).

⁸ See Frisch (1965).

⁹ See Hanoch (1975).

We now examine the relation between paths and the curves with equation

$$x_2^{1/k_2} = \theta x_1^{1/k_1}. \quad (2.35)$$

A comparison between (2.33) and (2.35) reveals that the expansion paths do not coincide with the “parabolic” curves of equation (2.35). However, if we define

$$\theta = \left[\frac{ak_2(k_1 - K)}{\beta k_1(k - k_2)} \right]^{k_2}$$

then the “parabolic” curves (2.35) coincide with the Wicksell curve (2.34).

If instead of (2.35) we consider the “parabolic” curve

$$x_2^{(1-k_2)/k_2} = \theta^* x_1^{(1-k_1)/k_1} \quad (2.36)$$

with

$$\theta^* = \left(\frac{\alpha k_2 p_2}{\beta k_1 p_1} \right)^{k_2/(1-k_2)}$$

then the expansion paths (2.33) coincide with the parabolic curves (2.36).

2.4.4 Elasticity of substitution

If the production function is of the form

$$y = f(x_1, x_2)$$

then the elasticity of substitution, σ , according to Hicks¹⁰ is a measure of the ease at which the varying factor can be substituted for others. It is defined as the proportional change in the factor ratio in response to a proportional change of the rate of substitution between the two factors. Symbolically, it is written as

$$\sigma = \frac{d(x_2/x_1)/(x_2/x_1)}{d(f_1/f_2)/(f_1/f_2)}.$$

It has been shown by Allen¹¹ that the above expression reduces to

$$\sigma = - \frac{f_1 f_2 (x_1 f_1 + x_2 f_2)}{x_1 x_2 [f_{11} f_2^2 - 2 f_1 f_2 f_{12} + f_{22} f_1^2]}$$

¹⁰ See Hicks (1963).

¹¹ See Allen (1960).

or

$$\sigma = - \frac{(f_1/x_2) + (f_2/x_1)}{(f_2/f_1)f_{11} - 2f_{12} + (f_1/f_2)f_{22}}. \quad (2.37)$$

To simplify (2.37) we note some properties of almost homogeneous functions

$$\left. \begin{aligned} (k-k_1)f_1 &= k_1 x_1 f_{11} + k_2 x_2 f_{12} \\ (k-k_2)f_2 &= k_1 x_1 f_{12} + k_2 x_2 f_{22} \end{aligned} \right\} \Leftrightarrow \begin{aligned} f_{11} &= \frac{k-k_1}{k_1 x_1} f_1 - \frac{k_2 x_2}{k_1 x_1} f_{12} \\ f_{22} &= \frac{k-k_2}{k_2 x_2} f_2 - \frac{k_1 x_1}{k_2 x_2} f_{12} \end{aligned}. \quad (2.38)$$

Substituting (2.38) into (2.37) we get the elasticities of substitution for almost homogeneous production functions:

$$\sigma = \frac{k_1 k_2 f_1 f_2 (x_1 f_1 + x_2 f_2)}{k_1 k_2 f_1 f_2 (x_1 f_1 + x_2 f_2) - k^2 f (f_1 f_2 - f f_{12})} \quad (2.39)$$

or

$$\sigma = \frac{1}{1 - \frac{k^2 f (f_1 f_2 - f f_{12})}{k_1 k_2 f_1 f_2 (x_1 f_1 + x_2 f_2)}}. \quad (2.40)$$

Obviously, when $k_1 = k_2 = 1$ then the elasticity of substitution (2.39) or (2.40) reduces to the standard one:

$$\sigma = \frac{f_1 f_2}{(k-1)f_1 f_2 + k f f_{12}}. \quad (2.41)$$

Substituting the partial derivatives of the almost homogeneous production function (2.16) into (2.39), the elasticity of substitution then becomes:

$$\sigma = \frac{\alpha k_2 x_1^{1/k_1} + \beta k_1 x_2^{1/k_2}}{\alpha (k_2 - 1) x_1^{1/k_1} + \beta (k_1 - 1) x_2^{1/k_2}}. \quad (2.42)$$

Therefore, the elasticity of substitution is not a constant, rather a function of the inputs x_1 and x_2 .

Thus, from (2.42), we see again that for convex isoquants we must have $k_1 > 1$ and $k_2 > 1$.

Suppose $k_1 = k_2 = 1$, then $\sigma \rightarrow \infty$.

Now, if $k_1 = k_2 = k^* \neq 1$ the elasticity of substitution from (2.42) reduces to a constant type elasticity of substitution, i.e., in the special case when

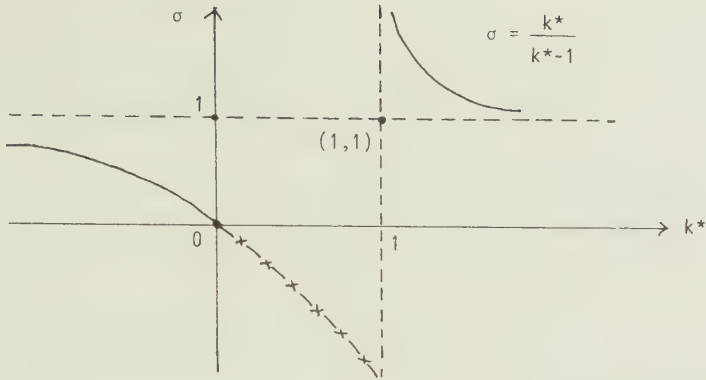


Fig. 2

$k_1=k_2=k^*\neq 1$ the AH production function (2.16) degenerates to a CES function, with:

$$\sigma = \frac{k^*}{k^*-1}. \quad (2.43)$$

If also $k_1=k_2=k^*<0$ then from (2.43) it follows that

$$\sigma > 0$$

but σ is a decreasing function of k^* , i.e.,

$$\frac{\partial \sigma}{\partial k^*} = -\frac{1}{(k^*-1)^2} < 0.$$

Graphically, in the case of (2.43) we have Fig. 2.

In the case where $0 < k^* < 1$ we have $\sigma < 0$, that is the iso-product curves are concave.

When $x_2^{1/k_2} = \theta x_1^{1/k_1}$ is given, the elasticity of substitution on this parabolic curve is

$$\sigma = \frac{\alpha k_2 + \beta k_1 \theta}{\alpha(k_2-1) + \beta(k_1-1)\theta}. \quad (2.44)$$

From (2.44) it follows that σ is constant along the curves with equation $x_2^{1/k_2} = \theta x_1^{1/k_1}$.

Let $\theta=0$, then from (2.44) we see that

$$\sigma = \frac{k_2}{k_2-1}$$

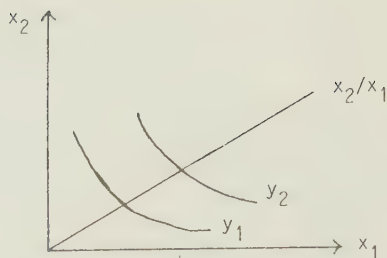


Fig. 3

also when $\theta \rightarrow \infty$, we have

$$\sigma = \frac{k_1}{k_1 - 1}.$$

And if $k_1 = k_2 = k^* \neq 1$ it is then seen that the elasticity of substitution, σ , is independent of the parameter θ and is given by

$$\sigma = \frac{k^*}{k^* - 1}.$$

2.5 Comparison of almost homogeneity and homotheticity

It is known that all homogeneous functions are homothetic, but not all homothetic functions are homogeneous. We have shown that every homogeneous function is also almost homogeneous.

So the question arises: can we show whether or not an almost homogeneous production function (2.16) is homothetic?

A production function $y = f(x_1, x_2)$ is homothetic¹² if it exhibits the following property:

$$\frac{\partial y / \partial x_1}{\partial y / \partial x_2} = \psi\left(\frac{x_2}{x_1}\right) = \psi(x), \quad x = \frac{x_2}{x_1}. \quad (2.45)$$

The property of homotheticity implies that the slope of an isoquant, $\psi(x)$, is a unique function of the slope of the ray through the origin (see Figure 3). The implication for the elasticity of substitution is that it is constant along a ray but not necessarily constant along each isoquant. In other words, this means that the property of homotheticity allows us to measure varying returns to scale production processes without making any assumptions about the elasticity of substitution.

¹² On this, see Wolkowitz (1971).

In the case of (2.16) we have:

$$\frac{\partial y / \partial x_1}{\partial y / \partial x_2} = \frac{\alpha k_2}{\beta k_1} \cdot \frac{x_1^{(1-k_1)/k_1}}{x_2^{(1-k_2)/k_2}} \neq \psi\left(\frac{x_2}{x_1}\right). \quad (2.46)$$

Therefore, the almost homogeneous production function (2.16) is not homothetic.

But when the parabolic curve $x_2^{1/k_2} = \theta x_1^{1/k_1}$ is given, then (2.46) yields:

$$\frac{\partial y / \partial x_1}{\partial y / \partial x_2} = \frac{\alpha k_2}{\beta k_1} \cdot \frac{x_2}{x_1} \cdot \frac{x_1^{1/k_1}}{x_2^{1/k_2}} = \frac{\alpha^k 2}{\beta \theta k_1} \cdot \frac{x_2}{x_1} = \psi\left(\frac{x_2}{x_1}\right). \quad (2.47)$$

Thus, along the "parabolic curves" the almost homogeneous production function (2.16) is becoming homothetic.

Therefore, such a function (2.16) is homothetic along each parabolic curve ($x_2 = \theta^{k_2} x_1^{k_2/k_1}$) in input space.

2.6 Pseudo-homotheticity

The most commonly used production functions, i.e. homogeneous functions like Cobb-Douglas and CES have constant scale elasticities. However, scale elasticity need not be constant for all levels of output.

According to Frisch's¹³ ultra passum law, scale elasticity decreases with the scale of operation. Thus, an attempt to test econometrically Frisch's hypothesis is not possible within the class of homogeneous production functions.¹⁴ In other words, this implies that in order to study the scale elasticity it is necessary to have more general production functions. Now, given the non-homothetic almost homogeneous production function

$$y = (\alpha x_1^{1/k_1} + \beta x_2^{1/k_2})^k \quad (2.16)$$

in this section we will show that when there is no reason to assume that k is constant a new type of production function can be derived having the same elasticity of substitution as (2.16) but different scale property.

That is, given the neoclassical production function (2.16) with a elasticity of substitution given by (2.42), it is possible to define a transformation

¹³ See Frisch (1965).

¹⁴ Ringstad (1974) presents an empirical application of inhomogeneous production functions. Ringstad also argues that "Although much of the theory of production is based on production functions with a scale elasticity that decreases with the scale of operation, very few attempts have been carried out to verify whether this is true or not."

which results in a generalized production function¹⁵ which has returns to scale that vary with output.

The definition of pseudo-homotheticity¹⁶ which is used is: a pseudohomothetic function or almost homothetic $y=g(x_1, x_2)$ is some non-decreasing monotonic transformation of an almost homogeneous function.

Formally we have:

$$y = g(x_1, x_2) = H[f(x_1, x_2)] \quad (2.48)$$

where $f(x_1, x_2)$ is an almost homogeneous function of x_1 and x_2 .

Since $y=H[f(x_1, x_2)]$ is an almost homogeneous function ($k_1, k_2, \alpha(y)$) and employing Euler's modified theorem, we have:

$$k_1 x_1 \frac{\partial H}{\partial x_1} + k_2 x_2 \frac{\partial H}{\partial x_2} = \alpha(y) \cdot y \quad (2.49)$$

We note that

$$\frac{\partial H}{\partial x_1} = \frac{\partial y}{\partial x_1} \quad (2.50.1)$$

$$\frac{\partial H}{\partial x_2} = \frac{\partial y}{\partial x_2} \quad (2.50.2)$$

$$\frac{\partial y}{\partial x_1} = \frac{dH}{df} \cdot \frac{\partial f}{\partial x_1} > 0 \quad (2.50.3)$$

$$\frac{\partial y}{\partial x_2} = \frac{dH}{df} \cdot \frac{\partial f}{\partial x_2} > 0 \quad (2.50.4)$$

By substitution from (2.50.1)–(2.50.4), equation (2.49) becomes:

$$\frac{dH}{df} \left[k_1 x_1 \frac{\partial f}{\partial x_1} + k_2 x_2 \frac{\partial f}{\partial x_2} \right] = \alpha(y) \cdot y. \quad (2.51)$$

In addition, we know $f(x_1, x_2)$ to be almost homogeneous of degree (k_1, k_2, k) and by Euler's modified theorem, we have

$$k_1 x_1 \frac{\partial f}{\partial x_1} + k_2 x_2 \frac{\partial f}{\partial x_2} = kf. \quad (2.51.1)$$

¹⁵ This is essentially the strategy followed by Zellner and Revankar (1969) in their analysis of generalized production functions.

¹⁶ See Al Ayat and Färe (1979), Kolm (1976).

Substituting for (2.51.1) in (2.51) and noting $dH/df=dy/df$, equation (2.51) can then be rewritten as a differential equation

$$\frac{dy}{df} = \frac{y\alpha(y)}{fk}. \quad (2.52)$$

This equation (2.52) has been derived by Zellner and Revankar. Thus, we have proved the following Zellner-Revankar theorem but in the case of almost homogeneous production functions:

Theorem. *Let $f(x_1, x_2)$ be an almost homogeneous production function of degree (k_1, k_2, k) . Then a production function $y=H[f(x_1, x_2)]$, with preassigned returns of scale function $\alpha(y)$, can be obtained by solving the following differential equation:*

$$\frac{dy}{df} = \frac{y}{f} \cdot \frac{\alpha(y)}{k}. \quad (2.52)$$

Førsund¹⁷ also has proved equation (2.52) by using the passus law. Therefore, the property of pseudo-homotheticity can be used to derive a numerical measure of variable returns to scale. This implies that by choosing the appropriate H transformation, we are capable of presenting a production process with variable returns to scale. Hence, we have proved that the Zellner-Revankar approach can be applied when f is an almost homogeneous function.

Furthermore, by specifying $\alpha(y)$, an almost homothetic production function can be generated by solving the differential equation (2.52). If for example we choose the function¹⁸

$$\alpha(y) = \frac{d}{1+by} \quad (2.53)$$

and substitute (2.53) into the differential equation (2.52) and specify f to be the almost homogeneous production function of type (2.16), then we have:

$$ye^{by} = (\alpha x_1^{1/k_1} + \beta x_2^{1/k_2})^d.$$

Such functions are difficult to estimate. There are few empirical tests of generalized production functions¹⁹ in the empirical production literature.

¹⁷ Førsund (1974).

¹⁸ If $b>0$, then the returns to scale fall from d , at $y=0$, to zero as $y \rightarrow \infty$. If $b<0$, $\alpha(y)$ increases from d , at $y=0$, to a positive constant, and approaches infinity as $y \rightarrow -1/b$. Obviously, if $b=0$ then $\alpha(y)=d$ which is a constant.

¹⁹ For a detailed empirical discussion, see F. Førsund and L. Hjalmarsson (1978).

2.7 A Box-Cox non almost homogeneous production function

The almost homogeneous production function (2.16) will now be extended in this section in order to include a constant term, using a generalization of the method of power transformation of variables developed by Box and Cox.²⁰

The power transformation introduced by Box and Cox is given as:

$$y^{(\gamma)} = \begin{cases} \frac{y^\gamma - 1}{\gamma}, & \gamma \neq 0 \\ \log y, & \gamma = 0 \end{cases} \quad (2.54)$$

where $y^{(\gamma)}$ expresses the transformation of the variable y . The method of Box-Cox applies to dependent variables as well as to independent variables. Zarembka²¹ notes that "... The transformation of variables may lead to a more nearly linear model, may stabilize the error variance, and/or may lead to a model for which a symmetrically, perhaps normally distributed error term is acceptable ... the Box-Cox procedure is robust to normality as long as the error term is reasonably symmetric ..."

Transformed variables can be included in an almost homogeneous function (2.16) so that generalized models of the form

$$y^{(\gamma)} = \alpha x_1^{(\gamma_1)} + \beta x_2^{(\gamma_2)} \quad (2.55)$$

can be specified.

Note that if $\gamma = 1$ in (2.55), then $y^{(1)}$ enters the equation linearly; also $y^{(0)}$ enters the equation as $\log y$, and $y^{(-1)}$ enters as the reciprocal of y .

Using (2.54) and

$$x_i^{(\gamma_i)} = \begin{cases} \frac{x_i^{\gamma_i} - 1}{\gamma_i}, & \gamma_i \neq 0 \\ \log x_i, & \gamma_i = 0 \end{cases}$$

equation (2.55) is given as:

$$\frac{y^\gamma - 1}{\gamma} = \alpha \frac{x_1^{\gamma_1} - 1}{\gamma_1} + \beta \frac{x_2^{\gamma_2} - 1}{\gamma_2}. \quad (2.56)$$

²⁰ See Box-Cox (1962). Note that the flexibility of the Cox-Box transformation has made it attractive for an increasing number of applications, for instance in monetary economics (Zarembka, 1968; Spitzer, 1976, 1977), income analysis (Heckman and Polachek, 1974) production theory (Appelbaum, 1979; Berndt and Khaled, 1979) and transportation (Hollyer et al., 1979).

²¹ See Zarembka (1975).

Table 1. *Alternative functional forms*

Value of γ	Value of γ_1	Value of γ_2	Functional form	Notes
$\gamma \rightarrow 0$	$\gamma_1 \rightarrow 0$	$\gamma_2 \rightarrow 0$	$\ln y = \alpha \ln x_1 + \beta \ln x_2$	Cobb-Douglas
$\gamma = 1$	$\gamma_1 = 1$	$\gamma_2 = 1$	$y = (1 - \alpha - \beta) + \alpha x_1 + \beta x_2$	Linear
$\gamma \rightarrow 0$	$\gamma_1 = 1$	$\gamma_2 = 1$	$\ln y = -(\alpha + \beta) + \alpha x_1 + \beta x_2$	Semi Logarithmic
$\gamma = 1$	$\gamma_1 \rightarrow 0$	$\gamma_2 \rightarrow 0$	$y - 1 = \alpha \ln x_1 + \beta \ln x_2$	Linear Logarithmic
$\gamma = \gamma_1 = \gamma_2 = \mu$	$\gamma = \gamma_1 = \gamma_2 = \mu$	$\gamma = \gamma_1 = \gamma_2 = \mu$	$y^\mu = (1 - \alpha - \beta) + \alpha x_1^\mu + \beta x_2^\mu$	Non-homogeneous CES ^a
$\gamma = 1$	$\gamma_1 = 2$	$\gamma_2 \rightarrow 0$	$y = \left(1 - \frac{\alpha}{2}\right) + \frac{\alpha}{2} x_1^2 + \beta \ln x_2$	
$\gamma = 1$	$\gamma_1 = 2$	$\gamma_2 = 2$	$y = \left(1 - \frac{a}{2} - \frac{b}{2}\right) + \alpha' x_1^2 + \beta' x_2^2$	Quadratic
$\gamma = 1$	$\gamma_1 = -1$	$\gamma_2 = -1$	$y = (1 + \alpha + \beta) - \alpha \frac{1}{x_1} - \beta \frac{1}{x_2}$	Reciprocal
$\gamma = \frac{1}{\alpha/\gamma_1 + \beta/\gamma_2}$	γ_1	γ_2	$y = \alpha \gamma x_1^{\gamma_1} + \beta \gamma x_2^{\gamma_2}$	Almost Homogeneous
$\gamma \rightarrow 0$	$\gamma_1 = 1$	$\gamma_2 = -1$		Ramsey Exponential ^b

^a Zarembka, P.: "Functional Form in the Demand for Money", Journal of the American Statistical Association, 1968, pp. 502-511.

^b Ramsey, I.: "Limiting Functional Forms for Demand Functions: Tests of Some Specific Hypotheses", Review of Econ. and Stat., 1974, pp. 468-477.

This generalized functional form contains as special cases the most well known functional forms:

- (i) if $\gamma = \gamma_1 = \gamma_2 = 1$ then we have the linear form;
- (ii) if $\gamma = \gamma_1 = \gamma_2 = 0$ then we have the Cobb-Douglas form;
- (iii) if $\gamma = \gamma_1 = \gamma_2 = \mu$ then we have the non-linear homogeneous CES form;
- (iv) if $\left. \begin{array}{l} \gamma = 1, \gamma_1 = \gamma_2 = 0 \\ \gamma = 0, \gamma_1 = \gamma_2 = 1 \end{array} \right\}$ then we have the semilog form.

These cases are given in Table 1.

It is well known that neither a priori reasoning nor economic theory clearly provides the correct functional form. The advantage with Box-Cox transformations is that the functional form is dictated by the parameters γ , γ_1 and γ_2 . By employing the Box-Cox transformations we have increased the range of functional form.

Several different versions of (2.55) are possible. Each specification places certain restrictions on γ 's.

Obviously, allowing for different values of γ 's a non-almost homogeneous production function can be obtained as:

$$y^\gamma = \left(1 - \frac{\alpha\gamma}{\gamma_1} - \frac{\beta\gamma}{\gamma_2}\right) + \alpha\gamma x_1^{\gamma_1} + \beta\gamma x_2^{\gamma_2}$$

or

$$y^\gamma = \alpha_0 + \alpha_1 x_1^{\gamma_1} + \beta_1 x_2^{\gamma_2} \quad (2.57)$$

or

$$y^{1/k} = \alpha_0 + \alpha_1 x_1^{1/k_1} + \beta_1 x_2^{1/k_2}. \quad (2.58)$$

By scaling the variables x_1 and x_2 , the non almost homogeneous production function (2.58) derived from Box-Cox transformations can be written as follows:

$$y^{\frac{1}{k}} = \alpha_0 + \lambda^{1/k} (\alpha_1 \bar{x}_1^{1/k_1} + \beta_1 \bar{x}_2^{1/k_2}) \quad (2.59)$$

where

$$\frac{x_1}{\bar{x}_1} = \lambda^{k_1/k}, \quad \frac{x_2}{\bar{x}_2} = \lambda^{k_2/k}$$

and $\bar{x}_1, \bar{x}_2$ are the sample arithmetic means and

$$\bar{y}^{1/k} = \alpha_1 \bar{x}_1^{1/k_1} + \beta_1 \bar{x}_2^{1/k_2}.$$

From (2.59) it follows that

$$\frac{1}{k} y^{1/k-1} \cdot \frac{\partial y}{\partial \lambda} = \frac{1}{k} \lambda^{1/k-1} (\alpha_1 \bar{x}_1^{1/k_1} + \beta_1 \bar{x}_2^{1/k_2})$$

or

$$\frac{\partial y}{\partial \lambda} \cdot \frac{\lambda}{y} = y^{-1/k} \bar{y}^{1/k} \cdot \lambda^k = \lambda^{1/k} \quad (\text{evaluated } y \text{ at } \bar{x}_1, \bar{x}_2).$$

But from (2.59) we have

$$\bar{y}^{1/k} = \alpha_0 + \lambda^{1/k} \bar{y}^{1/k}$$

or

$$\lambda^{1/k} = 1 - \frac{\alpha_0}{\bar{y}^{1/k}} = \varepsilon_\lambda. \quad (2.60)$$

It is easily seen from (2.60) that the following rule applies: if we consider the case where $a_0=0$ then $\varepsilon_\lambda=1$ and we have constant returns to scale. It further follows from (2.60) that $\varepsilon_\lambda>1$ when $a_0<0$; and if we assume that $a_0>0$, we are dealing with the case of decreasing returns to scale.

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Estimation of a Joint Cobb-Douglas and Almost Homogeneous Labour Function

“... Until the laws of thermodynamics are repealed, I shall continue to relate outputs to inputs—i.e. to believe in production functions.”

P. Samuelson

3.1 Introduction

In this chapter we seek to study the most fundamental characteristics (output elasticities, elasticities of substitution, returns to scale, etc.) of the production process in Swedish manufacturing industries, with the approach discussed in Chapter 2. These industries will be examined at the 2-digit level for the period 1963–1980.

In this study we have adopted a disaggregate approach. This implies that there has been a marked preference to analyze the production characteristics of Swedish manufacturing industries at very high levels of disaggregation. Otherwise we would have to use Bentzel's¹ procedure, e.g. to make the usual aggregation analysis and then to supplement it with a discussion of the effects of various components which enter into the aggregation.

Analysis at a high level of disaggregation has the advantage of enhancing the possibility of providing more numerous insights than analysis at a high level of aggregation. As Nadiri² observes: “... To evaluate correctly the role of technological change, it is necessary to formulate an alternative system of accounting which registers the utilities from different types of assets not only the conventional inputs, but also factors such as natural resources, physical environment, human skill and knowledge, social and political structures, etc. Such a conceptual framework would be able to accomodate both the purely technical advancement and the social innova-

¹ For detailed discussion, see Bentzel (1956).

² For this and other issues, see Nadiri's excellent survey (1970).

tions necessary to adopt the new technology. This approach requires concentration on more disaggregative studies such as microeconomic production functions, inter-industry resource allocation models, location theories, etc.”

In the analysis of production many issues arise. A first issue is the choice between a cost or a production function. The choice of the specific functional form to describe the input–output relationship is a problem which must be faced in any empirical production analysis. According to Goldberger³ “there is no magic rules which will tell us which the appropriate model to employ for a given empirical problem”. However, Fuss, McFadden, and Mudlak⁴ list the following five criteria upon which to judge the suitability of functional form:

- (i) Parsimony in parameters
- (ii) Ease of interpretation
- (iii) Computational ease
- (iv) Interpolative robustness within sample
- (v) Extrapolative robustness outside sample

A second issue is the choice of the appropriate functional form. The selection of the particular form depends on the problem under study. For example, if we want to examine the returns to scale then Kmenta’s approximation gives reliable estimates of this parameter. But if we want to examine the elasticity of substitution between capital and labour, then Kmenta’s approximation is inappropriate. Maddala and Kadane⁵ have shown, using Monte-Carlo techniques, that Kmenta’s procedure does not give reliable estimates of elasticity of substitution although it gives reliable estimates of the returns to scale parameter.

A third issue is whether technological progress should be treated as exogenous or endogenous, i.e. the problem of specification and representation of technological progress.

A fourth issue is concerned with the statistical estimation of the parameters of the production function. The fifth issue focuses on the specification of the stochastic error term. Little is known about how the stochastic error term in general should be introduced into a production function, although Zellner, Kmenta and Dréze⁶ in their pathbreaking paper have proposed one assumption that is compatible with the Cobb-Douglas production function.

The sixth issue is that, if there are more than two inputs, it is not clear how the elasticity of substitution should be defined since there is no unique

³ See Goldberger (1968 *a*).

⁴ See Fuss, McFadden and Mudlak (1978).

⁵ For a discussion of this result, see Maddala and Kadane (1968).

⁶ For detailed discussion, see Zellmer, Kmenta and Dréze (1966).

elasticity of substitution. Mundlak⁷ has shown that the different concept of elasticities of substitution are represented by different combinations of the underlying Hessian matrix.

The seventh issue is concerned with whether a set of different components in a sub-function can be aggregated into a single measure.

In empirical production theory, continuing emphasis is placed on the capital input, and little attention has been paid to the specification of the labour input. According to Berndt and Christensen: "... The aggregation of diverse capital inputs has drawn more criticism, most notably from the Cambridge school, than the aggregation of diverse labor inputs."⁸

Several studies have clarified some of these issues concerning the Swedish manufacturing industries. Most of these studies rely on the assumption that labour is a homogeneous input. It may be useful to explain more precisely the meaning of this assumption. Let us consider a hypothetical example. Take the case of the production function with two labour components, wage-earners (WE) and salaried-employees (SE) which form the aggregate input of labour (L), and one capital input (K) which is not an element of L .

Can one establish the existence of the aggregate labour input L ; i.e., $L=g(\text{WE}, \text{SE})$? Specifically, can we use wage-earners and salaried employees as components of an aggregate index of labour input? In other words, does the function $L=g(\text{WE}, \text{SE})$ exist?

The plan of this chapter is as follows: Section 3.2 discusses the single equation statistical models based upon alternative assumptions of firm behaviour. Several studies of Swedish manufacturing industries have employed the linear or direct aggregation, $L=\text{WE}+\text{SE}$, as a measure of labour input. The main disadvantage of this technique is the requirement of a priori knowledge that the value of substitution between WE and SE is infinite. As a result of the findings above, the purpose of section 3.3 is to specify the functional form of $g(\text{WE}, \text{SE})$. In section 3.4 we will outline the specification of both the production function and its stochastic error. Based on the results of the labour input specifications in the following section 3.5, two alternative production functions were estimated. In one case the stochastic error term is additive and in other it is multiplicative. The next step is to compare and discuss, from an econometric perspective, the additive and multiplicative models. This is of interest not only for the light it sheds on the stochastic error specification but also because the various elasticities arising from the two models can be different.

⁷ See Mundlak (1968).

⁸ See Berndt and Christensen (1974).

3.2 Three models of firm behaviour

The production function determines the maximum output given capital and labour inputs at a specified level of technology. If it is assumed that the industry chooses its decision variables—capital, labour and output—so as to maximize profits, then this implies that the production function alone does not determine the level of output. Instead, a system of equations (including profit considerations) will simultaneously determine the amount of capital, labour and output. Consequently, single equations estimation techniques applied to estimate the parameters of the production function could produce inconsistent estimates.

The purpose of this section is to present different production models that yield consistent estimates by use of a single equation technique.

By a consistent estimate we mean that the estimator of a given parameter has as its probability limit the actual parameter.

3.2.1 *Deterministic profit maximization*

Consider first the production function, which is taken to be Cobb-Douglas. Output is then defined as:

$$y_{it} = \alpha_0 x_{1it}^{\alpha_1} x_{2it}^{\alpha_2} \quad (3.1)$$

where

- y_{it} = production of the i th industry in year t
- x_{1it} = employment of the labour factor by the i th industry in year t
- α_0 = constant scale factor or a technology parameter
- α_1 = elasticity of output with respect to labour
- α_2 = elasticity of output with respect to capital.

Suppose that for each period of time, the firm employs the two inputs x_{1it} and x_{2it} so as to maximize its profits. This maximization goal leads to the following situation:

$$\left. \begin{aligned} \max \pi_{it} &= p_{it} y_{it} - w_{it} x_{1it} - r_{it} x_{2it} & i &= 1, \dots, n \\ y_{it} &= \alpha_0 x_{1it}^{\alpha_1} x_{2it}^{\alpha_2} & t &= 1, \dots, T \end{aligned} \right\} \quad (3.2)$$

which leads to the following first order profit maximizing conditions:

$$\frac{\partial L_{it}}{\partial y_{it}} = 0, \quad \frac{\partial L_{it}}{\partial x_{1it}} = 0, \quad \frac{\partial L_{it}}{\partial x_{2it}} = 0, \quad \frac{\partial L_{it}}{\partial \lambda} = 0$$

where L_{it} is the Lagrange function and λ the Lagrange multiplier. Utilizing these conditions it follows immediately that:

$$\alpha_1 \cdot \frac{y_{it}}{x_{1it}} = \frac{w_{it}}{p_{it}} \quad (3.3)$$

$$\alpha_2 \cdot \frac{y_{it}}{x_{2it}} = \frac{r_{it}}{p_{it}} \quad (3.4)$$

where p_{it} , w_{it} , r_{it} are respectively the price of output, the wage rate of labour input and the price of capital output.

The production function (3.1), with the equations (3.3) and (3.4), are the only outstanding relations which stand behind the optimal determination of both the level of inputs and output. The statistical model of production is an extension of the above—(3.1), (3.3) and (3.4)—economic model. The extension introduces a stochastic term into each relation. In its statistical form the model is written:

$$\begin{aligned} y_{it} &= \alpha_0 x_{1it}^{\alpha_1} x_{2it}^{\alpha_2} e^{u_{it}} \\ \alpha_1 \cdot \frac{y_{it}}{x_{1it}} &= \frac{w_{it}}{p_{it}} \cdot e^{\varepsilon_{1it}} \\ \alpha_2 \cdot \frac{y_{it}}{x_{2it}} &= \frac{r_{it}}{p_{it}} \cdot e^{\varepsilon_{2it}} \end{aligned} \quad (3.5)$$

where:

u_{it} is a stochastic term representing all the factors which cause the deviation of observed outputs from anticipated outputs;

ε_{1it} is a stochastic term representing the deviations and errors made by the firm in its attempts to adjust the level of x_{1it} input so as to satisfy the necessary condition for profit maximization;

ε_{2it} , same as ε_{1it} , but with respect to x_{2it} .

When expressed in logarithms (3.5) become in matrix form:

$$\begin{bmatrix} 1 & -\alpha_1 & -\alpha_2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \log y_{it} \\ \log x_{1it} \\ \log x_{2it} \end{bmatrix} + \begin{bmatrix} \log \alpha_0 & 0 & 0 \\ \log \alpha_1 & -1 & 0 \\ \log \alpha_2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ \log \frac{w_{it}}{p_{it}} \\ \log \frac{r_{it}}{p_{it}} \end{bmatrix} = \begin{bmatrix} u_{it} \\ \varepsilon_{1it} \\ \varepsilon_{2it} \end{bmatrix} \quad (3.6)$$

which may be rewritten as:

$$Mp + Nq = u \quad (3.7)$$

where

$$M = \begin{bmatrix} 1 & -\alpha_1 & -\alpha_2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad p = \begin{bmatrix} \log y_{it} \\ \log x_{1it} \\ \log x_{2it} \end{bmatrix}$$

$$N = \begin{bmatrix} -\log \alpha_0 & 0 & 0 \\ \log \alpha_1 & -1 & 0 \\ \log \alpha_2 & 0 & -1 \end{bmatrix} \quad q = \begin{bmatrix} 1 \\ \log \frac{w_{it}}{p_{it}} \\ \log \frac{r_{it}}{p_{it}} \end{bmatrix}$$

$$u = \begin{bmatrix} u_{it} \\ \varepsilon_{1it} \\ \varepsilon_{2it} \end{bmatrix}$$

The reduced form of (3.7) exists if

$$\text{Det } M = 1 - \alpha_1 - \alpha_2 \neq 0. \quad (3.8)$$

With constant returns to scale which implies $\text{Det } M = 0$, there exists no unique solution to this maximization problem. Sufficient conditions for unique profit maximization imply that $\text{Det } M = 1 - \alpha_1 - \alpha_2 > 0$. It is clear that the assumptions of perfect competition and profit maximization are inconsistent if $\text{Det } M \leq 0$, i.e. the production function exhibits a constant or increasing returns to scale.

Therefore, given that $\text{Det } M = 1 - \alpha_1 - \alpha_2 \neq 0$ the reduced form equation for the endogeneous variables of the production model (3.2) is:

$$p = -M^{-1}Nq + M^{-1}u. \quad (3.9)$$

In light of the above reduced form (3.9) it is easy to see that $\log x_{1it}$ and $\log x_{2it}$ are in fact not independent of u_{it} in the production function (3.5). Therefore, the least squares method (OLS) applied to the production function parameters would give inconsistent estimates of the parameters. Thus the OLS estimators are subject to a kind of simultaneous equation bias. Hence, there are both economic and statistical reasons to consider alternative techniques which take account of the stochastic nature of production in order to avoid the simultaneous equation bias.

We will first consider the reduced form estimation. If the variables in the vector q are exogeneous and independent of the stochastic error term, $M^{-1}u$, the OLS estimates of the reduced form parameters will be consistent. But this method of estimation would not yield unique estimates of the

production model since the system is not exactly identified. Also, it has been proved by Ringstad⁹ that this method is quite sensitive to measurement errors.

Therefore, the indirect least squares method is of little value in this framework.

Hoch¹⁰ has proposed a method of estimating Cobb-Douglas production functions without recognizing the role of the stochastic nature of the firm's decision. In other words, in spite of the fact that y_{it} , x_{1it} and x_{2it} are simultaneously determined, the equations describing the first order conditions are free from the stochastic term in the production function u_{it} . Also Kmenta¹⁰ has shown that the Hoch method is equivalent to indirect least squares.

Nerlove¹¹ used another method of reduced form estimation. He considers output as well as input prices as exogeneous variables and then shows by applying the well known Shepard's Duality Lemma that the cost function is the reduced form of the production model. According to Nerlove, consistent estimates of the cost function parameters yield consistent estimates of the dual production function parameters.

Thus, Nerlove has suggested that the parameters of production functions could be obtained by estimating a cost function, but because he ignores the stochastic nature of the firm's decision, this estimation technique becomes inappropriate for our purposes. There is a reason why Nerlove's model does not work in our framework. The reason is that output is treated as exogeneous in Nerlove's model and we must expect that output cannot be considered as exogeneous in our study. In general by using simultaneous equation techniques we can obtain consistent estimates of the production function parameters.

Jöreskog¹² has proposed an algorithm for the estimation of a linear system with linear constraints on the parameters. It is interesting to note that the Jöreskog procedure has been applied by Melander¹² with non-linear restrictions on the parameters. But consistent estimates of production function parameters can be obtained under some conditions, as shown below, by using a simple equation technique. The procedure for this problem is the specification of the error term. One possible specification of the character of the error term has been described by ZKD (Zellner, Kmenta and Dréze) and is also used by Feldstein¹³ and others.

The model of ZKD starts by asserting that the appropriate objective of a

⁹ See Ringstad (1971).

¹⁰ See Hoch (1958) and Kmenta (1964).

¹¹ See Nerlove (1965, 1967).

¹² See Jöreskog (1973) and Melander (1976).

¹³ See Feldstein (1971).

firm facing an uncertain production outcome is to maximize the mathematical expectation of profits. This objective function when applied to the Cobb-Douglas function¹⁴ has the property that the reduced forms of x_{1it} and x_{2it} are free from the stochastic term in the production function u_{it} .

The question which arises is: does the simultaneous equation technique perform better than the single equation technique? In terms of consistency, the two estimating techniques lead to consistent estimates.

In the extensive Monte Carlo analysis Guilkey and Lovell¹⁵ found "... the single equation estimating model to perform marginally better, at far less cost, than the system of equations model when the underlying technology is not translog".

Because of its relative simplicity, the single equation technique facilitates the analysis of the firm's behaviour and hence this approach is adopted in this thesis.

In the next section the ZKD model is presented. According to the ZKD, since $e^{u_{it}}$ is a random variable, output that depends on uncertainties regarding the quality of machinery also becomes a random variable. And because the profit function depends on output, profit becomes a random variable. Therefore, given the ZKD interpretation of the random variable $e^{u_{it}}$, the profit function for an industry is a random variable and obviously not deterministic as in the profit maximization model.

It will be seen in the next section that the implication that the profit function is a random variable is seen to be fundamental when single techniques were chosen over simultaneous equations techniques.

3.2.2 Maximization of expected profits or the ZKD model

According to Marschak and Andrews¹⁶ the production model can be presented as follows:

$$y_{it} = f(x_{1it}, x_{2it}) e^{u_{it}} \quad (\text{production function with a multiplicative error term})$$

$$\pi_{it} = p_{it} y_{it} - w_{it} x_{1it} - r_{it} x_{2it} \quad (\text{production function with } p_{it}, w_{it}, r_{it} \text{ predetermined})$$

$$\frac{\partial \pi_{it}}{\partial x_{1it}} = 0, \quad \frac{\partial \pi_{it}}{\partial x_{2it}} = 0 \quad (\text{first order conditions for profit maximization})$$

The interpretation of " u " is not clear in the literature, but Marschak and Andrews describe u as deviations between firms according to differences in

¹⁴ See Hodges, when the production function is of a CES type.

¹⁵ See Guilkey and Lovell (1980).

¹⁶ See Marchak and Andrews (1944).

“technical efficiency and depending in the technical knowledge, the will, effort and luck of a given entrepreneur”. Under such conditions the derived inputs are functions of the error term in the given equation, resulting in a violation of the basic assumptions of the Gauss-Markov theorem. In general, single equation estimation methods applied to production functions will give biased and inconsistent estimates when the inputs are correlated with error terms. This conclusion led to the development of various alternative estimation procedures based on different economic assumptions.

If the stochastic term in the production function could affect only the output and not the inputs, then there would be no simultaneous equation bias.¹⁷

Such an approach is in accordance with the requirement that each input is uncorrelated with the stochastic term in the production function in order to eliminate simultaneous equation bias. This requirement can be achieved by assuming that output is a random variable and that the decision unit maximizes the mathematical expectation of profit.

This approach has been discussed by Zellner, Kmenta and Dréze (ZKD). We follow the ZKD¹⁸ model which states:

An alternative specification, developed in the present paper, involves assuming that the production function of firm i is stochastic being defined by:

$$X_i = AL_i^{\alpha_1} K_i^{\alpha_2} e^{\mu_{0i}}$$

where μ_{0i} is a random disturbance representing factors such as weather, unpredictable variations in machine or labor performance and so on. Whenever the production process is not instantaneous the effect of the disturbance on output cannot be known until after the preselected quantities of inputs have been employed in production. Any given level of inputs will result in an uncertain quantity of output and, consequently, in an uncertain profit. The concept of “profit maximization” which is unambiguous within the deterministic framework of the model postulated by economic theory, needs a more subtle interpretation when stochastic elements are introduced; obviously no manager can maximize something which is uncertain and beyond his control.

If the production function is a Cobb-Douglas, the ZKD model shows that the reduced form equations for x_{1it} and x_{2it} are now independent of the stochastic error term. It is useful to present compactly the ZKD model.

The production function is given as:

$$y_{it} = \alpha_0 x_{1it}^{\alpha_1} x_{2it}^{\alpha_2} e^{u_{it}} \quad (3.10)$$

where $e^{u_{it}}$ is a stochastic error term which represents failures of machines,

¹⁷ For a detailed discussion, see Mundlak and Hoch (1965).

¹⁸ See Zellner, Kmenta and Dréze (1966).

variability of labour quality or according to ZKD "factors such as weather, unpredictable variations in machine or labour performance and so on".

Thus, the stochastic error term being analyzed in ZKD framework is of a different type from the stochastic error term of econometric analysis (measurement error, missing variables etc.).

In addition, u_{it} is normally distributed with zero mean constant variance σ^2 .

Zellner et al. assume that "(a) entrepreneurs maximize the mathematical expectation of profit; (b) the prices (p, w, r) (of output, x_{1it} and x_{2it} respectively) are either known with certainty or statistically independent of the production function disturbance, with expectations (p^+, w^+, r^+)".

Following the Zellner, Kmenta and Dréze assumption, the expected profit, $E(\Pi_i) = E(\Pi_i | x_{1it}, x_{2it})$, may be defined as

$$E(\Pi_i) = p^+ E(y_{it}) - w^+ x_{1it} - r^+ x_{2it}$$

where p^+, w^+ , and r^+ are the expectations for firm of p, w and r respectively.

The expected profit maximizing conditions are

$$\frac{\partial E(\Pi_i)}{\partial x_{1it}} = 0, \quad \frac{\partial E(\Pi_i)}{\partial x_{2it}} = 0. \quad (3.11)$$

Using the relations (3.10) and (3.11), we find that

$$\log y_{it} - \alpha_1 \log x_{1it} - \alpha_2 \log x_{2it} = \log \alpha_0 + u_{it} \quad (3.12)$$

$$\log y_{it} - \log x_{1it} = c_1 + u_{it} \quad (3.12.1)$$

$$\log y_{it} - \log x_{2it} = c_2 + u_{it} \quad (3.12.2)$$

where

$$c_1 = \log \left(\frac{w^+}{p^+ \alpha_1} \right) - \frac{1}{2} \sigma^2$$

$$c_2 = \log \left(\frac{r^+}{p^+ \alpha_2} \right) - \frac{1}{2} \sigma^2.$$

According to ZKD, we recognize that the maximizing conditions may not be exactly fulfilled because of managerial inertia, ignorance etc. Following ZDK we present these stochastic error terms as ε_{1it}^* and ε_{2it}^* for equations (3.12.1) and (3.12.2) respectively.

Furthermore, according to ZKD, we need to recognize deviations from optimality due to ex post differences between the expected and actual prices, ε_{1it}^+ and ε_{2it}^+ in the ZKD notation, so that

$$\log \left(\frac{w^+}{p^+} \right) = \log \left(\frac{w}{p} \right) + \varepsilon_{1it}^+ \quad (3.13.1)$$

$$\log \left(\frac{r^+}{p^+} \right) = \log \left(\frac{r}{p} \right) + \varepsilon_{2it}^+ \quad (3.13.2)$$

Substituting (3.13.1) and (3.13.2) into (3.12.1) and (3.12.2) and adding ε_{1it}^* and ε_{2it}^* we obtain

$$\log y_{it} - \alpha_1 \log x_{1it} - \alpha_2 \log x_{2it} = \log \alpha_0 + u_{it} \quad (3.12)$$

$$\log y_{it} - \log x_{1it} = c'_1 + u_{it} + \varepsilon_{1it} \quad (3.13.3)$$

$$\log y_{it} - \log x_{2it} = c'_2 + u_{it} + \varepsilon_{2it} \quad (3.13.4)$$

where

$$c'_1 = \log \left(\frac{w}{p} \right) - \frac{\sigma^2}{2}$$

$$c'_2 = \log \left(\frac{r}{p} \right) - \frac{\sigma^2}{2}$$

$$\varepsilon_{1it} = \varepsilon_{1it}^* + \varepsilon_{1it}^+$$

$$\varepsilon_{2it} = \varepsilon_{2it}^* + \varepsilon_{2it}^+$$

From (3.12), (3.13.3) and (3.13.4) we can solve for $\log x_{1it}$ and $\log x_{2it}$ to obtain reduced forms for $\log x_{1it}$ and $\log x_{2it}$. These reduced form equations are also contemporaneously independent of u_{it} giving us a sufficient condition which will allow the use of a simple equation technique.

In summary: with a Cobb-Douglas production function the ZKD model has shown that the reduced form equations for x_{1it} and x_{2it} are now independent of the stochastic term u_{it} . That is, the optimal input levels are not a function of u_{it} . If, furthermore, u_{it} is independent of ε_{1it} (and ε_{2it}), consistent estimates of the parameters of the Cobb-Douglas production function (3.12) are obtained directly by least squares.

The assumption of independence between u_{it} and the ε_{it} s has effectively separated the production function from the rest of expected profit maximizing conditions for estimation purposes. Hence, the single equation estimation techniques are still applicable.

The use of the multiplicative stochastic term requires an additional comment as far as parameter estimates are concerned. When a multiplicative error term is transformed logarithmically into a multiplicative lognormal stochastic error term for linear regression, attention is shifted from the conditional mean to the conditional median.

In as much as the conditional mean is the prime purpose of most empirical studies, Goldberger¹⁹ has advised that researchers report the value of the adjusted residual variance when linear logarithmic regressions are run so that results may be adjusted to avoid potential bias. However, Goldberger points out that in practice, the minimum variance unbiased estimators after adjustment may not differ substantially from those achieved by OLS, with no adjustment.

Grossman²⁰ has shown ZDK's results when expectations are rational. His view is summarized as follows: "... it is shown that rational expectations theory can give an economic justification for the use of ordinary least squares (OLS) to estimate the parameters of a Cobb-Douglas production function. ZDK justify the use of ordinary least squares in the context of an ad hoc model of expectations. Here we show that their result holds when expectations are rational."

In the model under discussion, the results above show that maximization of expected profits lead to consistent estimates of the parameters of the production function. But the assumption that firms attempt to maximize expected profits is consistent only with risk-neutral behaviour. According to Blair and Lusky²¹ (BL), "... if nonlinear attitudes towards risk are admitted, we must assume that firms attempt to maximize the expected utility of profits".

BL extended the ZDK model by assuming that the firms' optimization problem is defined by maximization of expected utility of profits, i.e. $\max E[U(\Pi)]$ where $U(\Pi)$ is a concave, continuous and differentiable function of profits with $U'(\Pi) > 0$ and $U''(\Pi) < 0$. Thus, $U(\Pi)$ is a utility function in the von Neumann-Morgenstern sense. With the assumption of maximizing the expected utility of profits, BL extended the analysis of ZDK to the case of a firm with nonlinear risk preferences. They demonstrated that: "... the Zellner-Kmenta-Dr  ze model is more robust than it indicates, i.e., its results can be generalized to include non-neutral risk preferences".

In the next section we can see that, when the parameters of the production function are random variables (assumption of technological uncertainty, see Feldstein) so that the firm does not know with certainty the parameters of the production function, and if the firm maximizes expected profits according to Zellner et al. then consistent estimates of the parameters of the production function are obtained by the single equation technique.

¹⁹ See Goldberger (1962). As noted by Goldberger when $y = \alpha_0 x_{1it}^{\alpha_1} \cdot x_{2it}^{\alpha_2} \cdot v$, where $v = e^u$, $u \sim N(\mu, \sigma^2)$, $E(v) = e^{1/2\sigma^2}$, Median = $M(v) = 1$; the systematic part of y_{it} obtained by deleting the stochastic error term, is thus the conditional median function rather than the conditional mean function.

²⁰ See Grossman (1975).

²¹ See Blair and Lusky (1975).

3.2.3 Maximization of expected profits under technological uncertainty: An extension

The ZKD model can be extended when the parameters α_0 and α_1 of the Cobb-Douglas production function (3.10) are random variables. Feldstein²² discussed the economic and econometric implication of production when uncertainty is incorporated into the production model. He assumed the following Cobb-Douglas production function

$$Y = \tilde{A} K^{\tilde{\beta}} L^{1-\tilde{\beta}} \tilde{V}$$

where Y is output, K is capital services, L is labour and $\tilde{V}$ is an independent random disturbance with $E(\log \tilde{V})=0$. $\tilde{A}$ and $\tilde{\beta}$ are fixed constants but are subjectively random in the Bayesian sense.

Two different maximization problems are investigated by Feldstein. The first maximization problem is that of profit maximizing and the second is that of maximizing expected output under a budget constraint. He considers that the enterprise has a utility of the form $U(Y)=cY^{\varrho}$ with $\varrho \leq 1$ or a constant elasticity demand function $p=DY^{\eta}$. He then considers that the distribution of $\alpha(=\log A)$ and β can be described as a bivariate normal. Thus the $E(y)$ can be evaluated by using the bivariate moment generating function (m.g.f.) for the random variables α and β . Then the first order conditions are derived:

$$p c (e^{\tilde{\alpha}} K^{\tilde{\beta}} L^{1-\tilde{\beta}})^{\varrho} \left[\exp \left(\frac{1}{2} \right) \varrho^2 \xi \right] K^{-1} [\tilde{\beta} + \varrho(x\sigma_{22} + \sigma_{12})] = \lambda r \quad (3.14)$$

$$p c (e^{\tilde{\alpha}} K^{\tilde{\beta}} L^{1-\tilde{\beta}})^{\varrho} \left[\exp \left(\frac{1}{2} \right) \varrho^2 \xi \right] L^{-1} [(1-\tilde{\beta}) - \varrho(x\sigma_{22} + \sigma_{12})] = \lambda w \quad (3.15)$$

where

$$E(\tilde{\alpha}) = \bar{\alpha}, \tilde{\beta} = E(\tilde{\beta}), \sigma_{11} = E(\tilde{\alpha} - \bar{\alpha})^2, \sigma_{22} = E(\tilde{\beta} - \bar{\beta})^2,$$

$$\sigma_{12} = E(\tilde{\alpha} - \bar{\alpha})(\tilde{\beta} - \bar{\beta}), x = \log \left(\frac{K}{L} \right), E(\exp \varrho \tilde{v}) = \exp \left[\frac{1}{2} \varrho^2 \sigma_{vv} \right]$$

$$\sigma_{vv} = E(\tilde{v}^2) \text{ and } \xi = [\sigma_{11} + x^2 \sigma_{22} + 2x\sigma_{12} + \sigma_{vv}].$$

The results are:

(i) According to Feldstein: "The customary procedure of replacing uncertain estimates by their mean values and solving the resulting non-stochastic problem does not yield the optimal input mix even in the absence of risk aversion ($\varrho=1$)."

²² See Feldstein (1971).

(ii) The optimal capital-labour ratio (X^*) is given as:

$$X^* = \frac{\bar{\beta} + \varrho(\sigma_{22}x^* + \sigma_{12})}{1 - \beta + \varrho(\sigma_{22}x^* + \sigma_{12})} \cdot \frac{w}{r} \quad (3.16)$$

where x^* is the log X^* .

From (16) it follows that "in the presence of uncertainty, the optimal capital labor ratio depends on the second moments of the production function parameters and not just the mean values".²³ The only exception is when the utility function has a constant elasticity of unity.

(iii) The stochastic error term ($\tilde{V}$) does not affect the optimal capital-labour ratio.

(iv) Production uncertainty has important implications for relative factor shares. It has been shown by Feldstein that the relative factor shares of the factors are altered.

(v) It should be observed and it can easily be proved that the expected output is not a homogeneous function.

(vi) Finally, because of the effects of technological uncertainty, the elasticity of substitution will not equal one, as in the nonstochastic case (certainty).

An objection can be raised to the formulation of constant returns to scale in Feldstein's model. But constant returns to scale in Feldstein's model imply that the random variables have the same variances, or in other words the correlation coefficient $\varrho_{\beta, 1-\beta}$ is equal to -1 . And this implies that if β increases then $1-\beta$ decreases. Therefore, I have introduced two plausible simplifications of Feldstein's model.

First, I assume that in the production function:

$$y_{it} = \alpha_0 x_{1it}^{\tilde{\alpha}_1} x_{2it}^{\tilde{\alpha}_2} v_{it} \quad (3.17)$$

α_0 is a non-random parameter. I have adopted this assumption because the randomness of α_0 does not affect the x^* . Thus, I consider the random variables $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ having a bivariate normal distribution $N_2(\tilde{\alpha}_1, \tilde{\alpha}_2, \sigma_{11}, \sigma_{22}, \varrho_{12})$ where $\tilde{\alpha}_1, \tilde{\alpha}_2$ and σ_{11}, σ_{22} are the mean and variances of $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ respectively, and ϱ_{12} is the correlation coefficient between them.

The second simplification concerns the returns to scale. From Feldstein's specification, the following question may arise: Can the relationship between the exponents of the inputs be described by constant returns to scale? We can show that the expected sum of the exponential coefficients is

²³ See Feldstein (1971).

an inappropriate measure of the returns to scale. Therefore, there is no need to assume returns to scale. In light of the above simplification it is clear that:

$$E(\bar{\alpha}_1) = \bar{\alpha}_1, E(\bar{\alpha}_2) = \bar{\alpha}_2, \sigma_{11} = E[(\bar{\alpha}_1 - \bar{\alpha}_1)^2] \\ \sigma_{22} = E[(\bar{\alpha}_2 - \bar{\alpha}_2)^2], \sigma_{12} = \text{cov}(\bar{\alpha}_1, \bar{\alpha}_2) = E[(\bar{\alpha}_1 - \bar{\alpha}_1)(\bar{\alpha}_2 - \bar{\alpha}_2)].$$

We assume, as in Feldstein's case, that the aggregate demand for production is given as:

$$p_{it} = c y_{it}^{n_0}.$$

Then, the optimal levels of factor inputs are determined by maximizing the expected profits $E(\Pi_{it})$, so, firms are maximizing expected profits instead of actual profits. The objective function is now:

$$\max E(\Pi_{it}) = E(\Pi_{it} | x_{1it}, x_{2it}) = E(p_{it} y_{it}) - w_{it} x_{1it} - r_{it} x_{2it}$$

subject to (3.17).

Max $E(\Pi_{it})$ requires the evaluation of $E[\alpha_0 x_{1it}^{\bar{\alpha}_1} x_{2it}^{\bar{\alpha}_2} v_{it}]^\mu$, where $\mu = \eta_0 + 1$, and assuming that the joint distribution of $\bar{\alpha}_1$ and $\bar{\alpha}_2$ is a bivariate normal distribution and u_{it} is log-normally distributed we obtain:

$$E[\alpha_0 x_{1it}^{\bar{\alpha}_1} x_{2it}^{\bar{\alpha}_2} v_{it}]^\mu = \alpha_0^\mu E[x_{1it}^{\bar{\alpha}_1} x_{2it}^{\bar{\alpha}_2}]^\mu \cdot E(v_{it}^\mu) \\ = \alpha_0^\mu E[x_{1it}^{\mu \bar{\alpha}_1} x_{2it}^{\mu \bar{\alpha}_2}] e^{\frac{1}{2} \mu^2 \sigma_{vv}} \\ = \alpha_0^\mu E[\exp(\mu \bar{\alpha}_1 \log x_1 + \mu \bar{\alpha}_2 \log x_2)] e^{\frac{1}{2} \mu^2 \sigma_{vv}} \\ = \alpha_0^\mu \exp \left\{ (\mu \log x_1) \bar{\alpha}_1 + (\mu \log x_2) \bar{\alpha}_2 + \frac{1}{2} \mu^2 [(\log x_1)^2 \sigma_{11} \right. \\ \left. + (\log x_2)^2 \sigma_{22} + 2(\log x_1)(\log x_2) \sigma_{12}] \right\}$$

therefore

$$E(\Pi_{it}) = c \alpha_0^\mu (x_1^{\bar{\alpha}_1} x_2^{\bar{\alpha}_2})^\mu \cdot e^{\frac{1}{2} \mu^2 \xi} - w_{it} x_{1it} - r_{it} x_{2it}$$

where

$$\xi = [(\log x_1)^2 \sigma_{11} + (\log x_2)^2 \sigma_{22} + 2(\log x_1)(\log x_2) \sigma_{12} + \sigma_{vv}].$$

The first order conditions then imply:

$$\alpha_0^\mu c (x_1^{\bar{\alpha}_1} x_2^{\bar{\alpha}_2})^\mu \mu e^{\frac{1}{2} \mu^2 \xi} \{ \bar{\alpha}_1 + \mu [(\log x_1) \sigma_{11} + (\log x_2) \sigma_{12}] \} x_{1it}^{-1} = \lambda w_{it} \\ \alpha_0^\mu c (x_1^{\bar{\alpha}_1} x_2^{\bar{\alpha}_2})^\mu \mu e^{\frac{1}{2} \mu^2 \xi} \{ \bar{\alpha}_2 + \mu [(\log x_2) \sigma_{22} + (\log x_1) \sigma_{12}] \} x_{2it}^{-1} = \lambda r_{it}.$$

Writing x^* for the optimal capital-labour ratio (x_{2it}^*/x_{1it}^*) we obtain

$$x^* = \left(\frac{x_{2it}^*}{x_{1it}^*} \right) = \frac{\bar{\alpha}_1 + \mu[(\log x_1) \sigma_{11} + (\log x_2) \sigma_{12}]}{\bar{\alpha}_2 + \mu[(\log x_2) \sigma_{22} + (\log x_1) \sigma_{12}]} \cdot \frac{w_{it}}{r_{it}}. \quad (3.18)$$

From (3.18) it follows that the ratio of the optimal input levels are independent of v_{it} .

If $\sigma_{12}=0$, and since we have a bivariate normal distribution, then $\bar{\alpha}_1$ and $\bar{\alpha}_2$ are independently distributed. Also, the optimal capital-labour ratio in this case ($\sigma_{12}=0$) is written as:

$$x^* = \frac{\bar{\alpha}_1 + \mu \sigma_{11} \log x_{1it}}{\bar{\alpha}_2 + \mu \sigma_{22} \log x_{2it}} \cdot \frac{w_{it}}{r_{it}}$$

which implies that even if $\sigma_{12}=0$, the optimal capital-labour ratio depends on second moments of production.

Now, we are going to discuss the question: Does the sum of the exponentials represent returns to scale? From Euler's theorem it follows that

$$x_1 \frac{\partial E(y)}{\partial x_1} + x_2 \frac{\partial E(y)}{\partial x_2} = E(y)$$

and in the case of Cobb-Douglas with random exponential coefficients we observe that:

$$\bar{\alpha}_1 + \bar{\alpha}_2 + \mu \{ (\sigma_{11} + \sigma_{12}) \log x_1 + (\sigma_{22} + \sigma_{12}) \log x_2 \} = 1 \quad (3.19)$$

and when no uncertainty is assumed ($\sigma_{11}=\sigma_{22}=\sigma_{12}=0$) we have $\bar{\alpha}_1 + \bar{\alpha}_2 = 1$, as in the analysis under certainty.

The above results (3.19) indicate that the expected sum of the exponential coefficients is an inappropriate measure of the returns to scale, or in other words $\bar{\alpha}_1 + \bar{\alpha}_2$ underestimate the returns to scale.

Finally, if $\bar{\alpha}_1$ and $\bar{\alpha}_2$ are identically distributed, dependent random variables, then the sum of the exponential coefficients is less likely to be near normal than either $\bar{\alpha}_1$ and $\bar{\alpha}_2$. In other words, this implies that combining dependent random variables, $\bar{\alpha}_1$ and $\bar{\alpha}_2$ can produce a variable $\bar{\alpha}_1 + \bar{\alpha}_2$ that is non-normally distributed, i.e. is less near normal than its components.²⁴

Given the above modification of Feldstein's model, it is useful to derive its reduced form. From the first order conditions we have:

$$\frac{\mu \cdot E(y_{it}) \{ \bar{\alpha}_1 + \mu [\sigma_{11} (\log x_{1it}) + \sigma_{12} (\log x_{2it})] \}}{x_{1it}} = w_{it}. \quad (3.20)$$

²⁴ For a discussion of this result, see Granger (1976).

$$\frac{\mu \cdot E(y_{it}) \{ \bar{\alpha}_2 + \mu [\sigma_{12}(\log x_{1it}) + \sigma_{22}(\log x_{2it})] \}}{x_{2it}} = r_{it}. \quad (3.21)$$

According to ZKD the above equations (3.20, 3.21) may not be satisfied for two reasons. First is the notion that due to human error, the firm may not produce exactly at the point where (3.20, 3.21) is satisfied. Second, we must recognize that the relative input prices in (3.20, 3.21) are not known with certainty and following the ZKD approach see equations (3.13.1 and 3.13.2).

The ‘‘pseudo-reduced form’’ (pseudo in the sense that an explicit solution for each x_{it} is not given²⁵) of the model is then:

$$\left. \begin{aligned} \log y_{it} &= h(x_{1it}, x_{2it}) + \log v_{it} \\ \log E(y_{it}) + \log \varphi_1(x_{1it}, x_{2it}) &= \log \left(\frac{w_{it}}{p_{it}} \right) + \log v_{it} + \varepsilon_{1it} \\ \log E(y_{it}) + \log \varphi_2(x_{1it}, x_{2it}) &= \log \left(\frac{r_{it}}{p_{it}} \right) + \log v_{it} + \varepsilon_{2it} \end{aligned} \right\}$$

This ‘‘reduced’’ form is contemporaneously independent of v_{it} giving us a sufficient and necessary condition which will allow us to use a single equation technique. For the modified Feldstein model, OLS for the first equation of pseudo-reduced form will yield consistent estimates of the production function parameters.

In conclusion, equation (3.17) has an important implication for the estimation problem. It reveals that in setting the first order conditions for the expected profit maximization the firm does not consider v_{it} itself but the expected value of v_{it} . As a result, factors which appear in v_{it} do not influence the inputs. Consequently, the determination of factor input levels is independent of the randomness of v_{it} and a single equation technique may be appropriate under certain circumstances.

3.3 Labour input as an almost homogeneous function

In most production function studies two aggregated categories of inputs are considered: capital and labour. The focus of the majority of these studies have been to determine the elasticity of substitution between capital and labour. In addition to the problem of estimating the elasticity of substitution between capital and labour, an important problem is how persons with different levels of education should be aggregated to a measure of total

²⁵ For a detailed analysis of this form, see De Janvry (1972).

labour input. We must move beyond the two factor production functions into a more general model which allows us to examine the elasticity of substitution between different types of labour and different types of capital. However, two main issues arise here that are important in production theory. The first issue is how to aggregate the labour or capital input. The second issue refers to situations where there are more than two inputs and it is not clear how the elasticity of substitution should be defined since there is no unique definition of the elasticity of substitution.²⁶ In labour aggregation three approaches have been used in constructing an aggregated labour input.

The first is the simple linear form:

$$L = L_1 + L_2 + \dots + L_n \quad (3.22)$$

In this case the elasticity of substitution between each pair of $L_1, \dots, L_n$ is equal to infinity. This aggregate labour input is the most convenient form.

In the second approach, L is a weighed sum of different types of labour with the weights being the relative wages:

$$L = L_1 + w_2 L_2 + \dots + w_n L_n. \quad (3.23)$$

Also in this case the elasticity of substitution between each pair of $L_1, \dots, L_n$ is equal to infinity.

In the third approach a CES function is adopted in order to formulate the aggregate labour input:

$$L = (a_1 L_1^{-p} + a_2 L_2^{-p} + \dots + a_n L_n^{-p})^{-1/p}. \quad (3.24)$$

The third approach has been utilized by Bowles²⁷ and Dougherty.²⁸

Bowles classified labour into three components: persons with less than eight, eight to eleven, and more than eleven years of schooling. He then used employment and wage data from twelve countries to estimate the elasticity of substitution between each pair of components.

Bowles found that the elasticity of substitution between the higher two labour components was not significantly different from infinity, and hence they could be aggregated linearly into a single component.

Dougherty used the Bowles framework to estimate pairs of elasticities of substitution between the labour components. In his study, Dougherty used two classifications of labour input, one defining components by occupation and the other defining them by educational level. Dougherty found "...

²⁶ See, for example, recent work by Fuss and McFadden (1978).

²⁷ See Bowles (1970).

²⁸ See Dougherty (1972).

that (the elasticities of substitution) were always significantly higher than unity at the 1 percent level but generally significantly lower than infinity at the 5 percent level”.

In his paper, Tinbergen²⁹ suggests that the parameters of the Bowles and Dougherty models were too high. According to Tinbergen’s analysis, the overestimation of the parameters resulted from a specification error of the model, i.e., only the demand side is specified whereas the supply side is left out of consideration. In particular, he concludes that the elasticities of substitution between different components of labour are closer to one (the Cobb-Douglas case).

Groenveld and Kuipers³⁰ reported on some tests of Tinbergen’s analysis. Their empirical results suggest that Tinbergen’s results are highly sensitive to the exogenous variables chosen and to the specification of the demand and supply equations. They conclude that “... there is hardly any reason to argue that the elasticity of substitution is equal to one”.

The main motivation behind our specification of the labour input is derived from two inadequacies of the CES framework used by Bowles and Dougherty. They are: (i) the procedure of aggregating the labour input when the number of labour components is greater than two and (ii) the treatment of the elasticity of substitution between two labour components.

Consider (i) first. It is well-known that if the number of labour components is greater than two, then all the pairwise elasticities of substitution are equivalent in the context of the CES function. It follows, therefore, that the CES specification of the labour input is inappropriate when the number of labour components is greater than two.

Thus, we shall search for functional forms of the labour input which can attain an arbitrary set of elasticities of substitution. It can be shown³¹ that the almost homogeneous function defined in chapter 2 can attain an arbitrary set of such elasticities. The treatment of elasticity of substitution in the case of two labour components is the second problem. In the case of two labour components, the CES labour specification constraints the elasticity of substitution to be a constant so that it is not allowed to vary. We have shown in Chapter 2 that an almost homogeneous function defined in a two input space allows the elasticity of substitution to be variable.

In this section we attempt to overcome the above mentioned problem (ii) of the CES framework. Our aim will be to bring out the statistical link between the occupational structure of the aggregate labour input and the almost homogeneous function.

²⁹ See Tinbergen (1974).

³⁰ See Groenveld and Kuipers (1976).

³¹ See Mukerji (1963).

Now, let

$$L = f(\text{WE}, \text{SE})$$

be the labour input function connecting wage earners (WE) and salaried employees (SE).

The issue we want to concentrate on here is the measurement of L , or in other words to find out the specification of $f(\text{WE}, \text{SE})$ from our data.

In the estimation of labour input, the researcher has several options available. First, the linear function (3.22) can be used. A similar approach is to estimate the weighed linear function (3.23). A second approach is to estimate the Cobb-Douglas function. A third approach is to estimate the CES function. In the third approach or according to Bowles-Dougherty's framework, the aggregate labour input is not a single summation of these two components or a price weighted summation, but a CES function:

$$L = [\delta(\text{WE})^{-\varrho} + (1-\delta)(\text{SE})^{-\varrho}]^{-1/\varrho}. \quad (3.25)$$

Now, we can use Sato's³² procedure in order to estimate the ϱ and δ 's. If we combine the first order conditions for competitive factor markets, namely

$$\frac{\partial L}{\partial \text{WE}} \bigg/ \frac{\partial L}{\partial \text{SE}} = \frac{p_w}{p_s} \quad (3.26)$$

where p_w is the price of WE and p_s is the price of SE, one gets the following expression:

$$\left(\frac{\text{WE}}{\text{SE}} \right)^{1+\varrho} = \left(\frac{\delta}{1-\delta} \right) \cdot \left(\frac{p_w}{p_s} \right)^{-1}. \quad (3.27)$$

From this expression, (3.27), we obtain the estimates of the ϱ and δ 's.

In the estimation of equation $L=f(\text{WE}, \text{SE})$, we desire a more general functional form that places a few a priori restrictions on the labour input function. The almost homogeneous function, discussed in Chapter 2, allows considerable generality, since it places no restriction on the elasticity of substitution and it includes as special cases the Cobb-Douglas and the CES functions.

Our almost homogeneous function³³ model appears never to have been estimated within the context of labour input.

³² See Sato (1967). Such general two stage formulation of production function have also appeared in Bergström (1982).

³³ It should be noted that the almost homogeneous function is not the only functional form offering such generality. For example, the translog offer similar generality.

We now proceed to describe the almost homogeneous function model.

We now assume that there exists a simple, almost homogeneous function of degree $k=1$ ³⁴, k_1, k_2 with two independent variables: WE and SE. This implies that the aggregate labour input could be written in the following form:

$$L = \delta(WE)^{1/k_1} + (1-\delta)(SE)^{1/k_2} \quad (3.28)$$

where L is an aggregation of labour.

We have from (3.28):

$$\frac{\partial L}{\partial WE} = \frac{\delta}{k_1} (WE)^{\frac{1}{k_1}-1} \quad (3.29)$$

$$\frac{\partial L}{\partial SE} = \frac{1-\delta}{k_2} (SE)^{\frac{1}{k_2}-1} \quad (3.30)$$

and since we assume perfect competition we have

$$\frac{\partial L}{\partial WE} = p_w \quad \frac{\partial L}{\partial SE} = p_s \quad (3.31)$$

using (3.29) and (3.30) in (3.31) we can obtain the following relations

$$\frac{\delta}{k_1} (WE)^{\frac{1}{k_1}-1} = p_w \quad (3.32)$$

$$\frac{1-\delta}{k_2} (SE)^{\frac{1}{k_2}-1} = p_s. \quad (3.33)$$

Now the left side of (3.32) and (3.33) can be written as

$$WE = \left(\frac{k_1}{\delta} \right)^{k_1/1-k_1} \cdot p_w^{k_1/(1-k_1)} \quad (3.34)$$

$$SE = \left(\frac{k_2}{1-\delta} \right)^{k_2/1-k_2} \cdot p_s^{k_2/(1-k_2)}. \quad (3.35)$$

³⁴ Observe that our objective is to estimate the aggregate labour input, L . Hence the assumption of $k=1$ in almost homogeneous function (3.28) has been reasonable. In order to estimate k_1 and k_2 we use the first order conditions for competitive factor markets. Observe that the equation (3.26) is independent of k . Hence the assumption $k=1$, does not affect the estimation procedure or nothing would be changed in the estimates if k were set at any other number. With k selected as above, a close relationship exists between k_1 and k_2 . Furthermore, with $k=1$ it then follows that k_1 and $k_2 > 1$ in order to have a well defined labour function (convex to the origin).

Using (3.34) and (3.35) to express the ratio $\frac{WE}{SE}$ as a function of prices we get

$$\frac{WE}{SE} = \frac{\left(\frac{k_1}{\delta}\right)^{k_1/(1-k_1)}}{\left(\frac{k_2}{1-\delta}\right)^{k_2/(1-k_2)}} \cdot \frac{p_w^{k_1/(1-k_1)}}{p_s^{k_2/(1-k_2)}} \quad (3.36)$$

The parameters of (3.28) were estimated by taking the logarithms of (3.36):

$$\log\left(\frac{WE}{SE}\right) = \alpha_0 + \alpha_1 \log p_w + \alpha_2 \log p_s \quad (3.37)$$

A different approach is to consider L as output and WE , SE as inputs. If the cost function is input-output separable so that

$$C = c_1(L) C_2(p_w, p_s)$$

then by Shephard's lemma, we have:

$$\frac{WE}{SE} = \frac{\partial C / \partial p_w}{\partial C / \partial p_s} = \frac{\partial C_2 / \partial p_w}{\partial C_2 / \partial p_s}$$

Define

$$C_2(p_w, p_s) = \alpha_1 p_w^\mu + \alpha_2 p_s^\nu$$

and by Shephard's lemma we obtain the relation (3.37)

where³⁵

$$\alpha_0 = \log \frac{\left(\frac{k_1}{\delta}\right)^{k_1/(1-k_1)}}{\left(\frac{k_2}{1-\delta}\right)^{k_2/(1-k_2)}} \quad (3.38)$$

³⁵ As Hamermesh and Grant (1979) point out, if labour is classified by its occupational (wage-earners and salaried employees), the wages should be treated as exogeneous. According to Hamermesh and Grant: "... If labor is disaggregated by age groups, a production function approach, or some method in which factor quantities are exogeneous, is the better choice: While supplies of some types of labor may respond to changes in factor prices, this is likely to be unimportant for many groups, so that on the net age structure of the disaggregation by occupation, historical changes in occupation wage differentials are consistent with relative supply elasticities to occupations that are very large. Accordingly, wages are more appropriately treated as exogeneous if labor is disaggregated by some occupational classification."

$$\alpha_1 = \frac{k_1}{1-k_1} \quad (3.39)$$

$$\alpha_2 = -\frac{k_2}{1-k_2}. \quad (3.40)$$

We can see that equation (3.37) reduces to Sato's equation (3.27) when

$$\alpha_1 = -\alpha_2. \quad (3.42)$$

To test the validity of the almost homogeneous labour input specification we employ an t -test procedure. The t statistic can be employed to test for values of any linear combination of regression coefficients.

We consider the null hypothesis

$$H_0: \alpha_1 + \alpha_2 = 0$$

and the alternative

$$H_1: \alpha_1 + \alpha_2 \neq 0.$$

So, to test H_0 against H_1 we just compute

$$t_0 = \frac{\hat{\alpha}_1 + \hat{\alpha}_2}{\sqrt{\text{Var}(\hat{\alpha}_1 + \hat{\alpha}_2)}}$$

where the variance, $\text{Var}(\hat{\alpha}_1 + \hat{\alpha}_2)$, is given by

$$\text{Var}(\hat{\alpha}_1 + \hat{\alpha}_2) = \text{Var}(\hat{\alpha}_1) + \text{Var}(\hat{\alpha}_2) + 2 \text{Cov}(\hat{\alpha}_1, \hat{\alpha}_2)$$

and we compare t_0 with the critical t value. H_0 is rejected in favour of H_1 if t_0 exceeds the critical t value.

If H_0 is rejected then our labour specification is given by (3.28). This means that we should use the almost homogeneous function (3.28) as a labour input.

If H_0 is maintained then we estimate the first step of Sato's procedure (see equation (3.43), i.e.

$$\log\left(\frac{\text{WE}}{\text{SE}}\right) = \alpha_0 + \alpha_1 \log\left(\frac{p_w}{p_s}\right) \quad (3.43)$$

where $\alpha_0 = \sigma \log\left(\frac{\delta}{1-\delta}\right)$ and $\alpha_1 = -\sigma$.

The CES labour function (3.43) has been estimated by many economists over the years. It is of interest to subject it to the same type of examination carried out for the almost homogeneous labour function (3.28).

To test the hypothesis that the CES labour function (3.25) is an appropriate functional form, the following hypothesis is tested

$$H_0: \sigma = 1$$

$$H_1: \sigma \neq 1.$$

If H_0 is rejected, then the CES labour function (3.25) is accepted as being an appropriate labour index.

If H_0 is not rejected, then the Cobb-Douglas labour function ($L = A(WE)^{\alpha_1}(SE)^{\alpha_2}$) is an appropriate functional form.

Finally, if $1/\alpha_1 = 1/\alpha_2 = 0$ we can write, using (3.39) and (3.40):

$$\frac{1-k_1}{k_1} = -\frac{1-k_2}{k_2} = 0 \quad (3.44)$$

as $k_1 \neq 0$ and $k_2 \neq 0$, we must have from (3.44)

$$k_1 = 1, k_2 = 1. \quad (3.45)$$

Therefore, the labour input has the linear form

$$L = \delta(WE) + (1-\delta)(SE). \quad (3.46)$$

Stated more formally, the hypotheses to be tested in this section are as follows:

Hypothesis 1: Variable Elasticity of Substitution between WE and SE. The labour input is given as an almost homogeneous function (3.28). The null hypothesis that $\alpha_1 = -\alpha_2$ was valid was tested by using a t -statistic.

If *Hypothesis 1* is correct, the null hypothesis ($H_0: \alpha_1 = -\alpha_2$) must be rejected.

Hypothesis 2: Constant Elasticity of Substitution between WE and SE. In the event that the *Hypothesis 1* is rejected, we then estimate the equation (3.27). The hypothesis that the labour input is a CES type (*Hypothesis 2*) was tested by estimating the equation (3.27).

If *Hypothesis 2* is correct, the coefficient of $\log(p_w/p_s)$ should be statistically different from zero and one.

Hypothesis 3: Unitary Elasticity of Substitution between WE and SE. If the hypothesis that the elasticity of substitution—see *Hypothesis 2*—is different from one is rejected, then the findings are consistent with the Cobb-Douglas labour specification. In this case the CES function reduces to the Cobb-Douglas function in which the elasticity of substitution equals 1.

Hypothesis 4: Infinite Elasticity of Substitution between WE and SE. This implies perfect substitution between the factors in question. If

Table 1. *Least squares estimates of the almost homogeneous function*

$\log \left(\frac{WE}{SE} \right) = \alpha_0 + \alpha_1 \log p_w + \alpha_2 \log p_s + u$							
Sample period 1963–1980							
ISIC	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{k}_1$	$\hat{k}_2$	R^2	D.W.
31	0.67988	-1.05897* (0.17938)	1.01998* (0.19124)	17.96**	51.05**	.773	1.12
32	1.17259	-0.55861* (0.08804)	-0.53631* (0.09795)	-1.27	3.49	.819	1.4
341	2.2728	-0.90161* (0.19015)	0.55984* (0.21783)	9.16	0.349	.98	1.7
342	0.39445	-1.68875* (0.23193)	1.54864* (0.26085)	2.45**	2.82**	.955	1.6
34	1.4538	-1.10343* (0.18521)	0.85050* (0.20752)	-10.6	-5.38	.98	1.7
35	0.93130	-0.07212 (0.06772)	-0.00370 (0.07746)	-0.078	0.004	.81	1.5
36	2.09183	-1.48055* (0.29978)	1.02302* (0.31074)	3.081**	44.44**	.947	2.1
37	1.27175	-0.54865 (0.50332)	0.43255 (0.51484)	-1.216	-0.762	.596	1.02
38	1.8445	-0.01831 (0.20630)	-0.23848 (0.20586)	0.018	0.193	.92	2.4

Note: Numbers in parentheses are the estimated standard errors of the respective coefficients.

* Denotes estimates significantly different from zero at the 0.05 level.

** The almost homogeneous function is well defined, i.e. $k_1 > 1$ and $k_2 > 1$.

$k_1 = k_2 = 1$, the almost homogeneous function (3.28) then degenerates into the linear function, $L = WE + SE$.

Therefore, the hypothesis testing is divided into two stages. In the first stage tests, the decision is made whether or not the elasticity of substitution is constant. If the elasticity of substitution is constant, then, in the second stage, the decision is made about which one of the two alternative models—CES and Cobb-Douglas—is the appropriate labour specification.

The hypotheses were tested by estimating two different regression equations—(3.37) and (3.43)—utilizing time series data.

The results of the ordinary least squares regression for the almost homogeneous labour specification—(3.37)—appear in Table 1.

In three cases—35, 37 and 38—where the inverse of the estimated coefficients of almost homogeneous function are not significantly different from zero, and an F -statistic results in the rejection of the almost homogeneous

Table 2. *Elasticities of substitution between wage-earners and salaried employees*

	ISIC 342	ISIC 36
	σ	σ
1963	1.58773	1.02595
1964	1.58772	1.02596
1965	1.58800	1.02595
1966	1.58846	1.02600
1967	1.58893	1.02606
1968	1.58943	1.02629
1969	1.58901	1.02617
1970	1.58980	1.02653
1971	1.59008	1.02671
1972	1.59051	1.02687
1973	1.59078	1.02691
1974	1.59079	1.02685
1975	1.59071	1.02684
1976	1.59108	1.02699
1977	1.59137	1.02714
1978	1.59163	1.02738
1979	1.59183	1.02747
1980	1.59198	1.02757

and leads to the acceptance of linear specification as an appropriate procedure, therefore, the hypothesis that the parameters k_1 and k_2 are equal to one must be accepted, which means that the factors, WE (wage-earners) and SE (salaried-employees) are perfectly substitutable.

Of the nine industries for which regression equations (3.28) are estimated we were able to reject the null hypothesis, $H_0: \alpha_1 + \alpha_2 = 0$, in just two industries (342 and 36). In the case of 342 the computed value of t_0 is 3.22 which is greater than the critical t -value. In the case of 36 the computed value of t is 13.44.

In this case the issue of primary interest is the elasticity of substitution between wage earners and salaried employees. Utilizing the result of Chapter 2, see equation (2.42), the estimation of the elasticities of substitution are reported in Table 2. The elasticities of substitution indicate a high degree of substitutability between wage earners and salaried employees.

For the remaining four—31, 32, 341, and 34—industries we shall now investigate if the CES Labour Function (3.25) is supported by the data.

Since we have only two labour components, this allowed us to use the first stage of the Sato model. This model is summarized in equation (3.43). In this situation it can be easily shown that the coefficient on the log of the

Table 3. *Least squares estimates of the Sato's first stage*

$\log \left(\frac{WE}{SE} \right) = \alpha_0 + \alpha_1 \log \left(\frac{p_u}{p_s} \right) + u$					
Sample period 1963–1980					
ISIC	$\hat{\alpha}_0$	$\hat{\alpha}_1$	σ	R^2	D.W.
31	0.49175	-1.15861* (0.17144)	1.15861	.74	1.2
32	1.06694	-0.61672* (0.07732)	0.61672	.80	1.4
341	1.14467	-0.56267* (0.26465)	0.56267	.87	1.2
34	-0.04411	-2.19584* (0.27250)	2.19584	.80	1.3

Note: Numbers in parentheses are the estimated standard errors of the respective coefficients.

* Denotes estimates significantly different from zero at the 0.05 level.

relative wages when it is regressed on the log of relative employment is a best, linear, unbiased estimate of the elasticity of substitution between wage earners and salaried employees.

Equation (3.43) was estimated for the industries 31, 32, 341, and 34. The results are shown in Table 3.

The coefficient on the log of relative wages had the expected negative sign and was significantly different from zero at the 5% level. The elasticity of substitution between wage earners and salaried employees implied by this coefficient is quite high for the industries 31 and 34.

Only in two out of four industries—32 and 34—is the estimated elasticity of substitution significantly different from unity. These results tend to support the Cobb-Douglas labour function— $L = (WE)^{\alpha_1} (SE)^{1-\alpha_1}$ —in two cases—31 and 341.

From the results examined above, the main result can be summarized as follows: the empirical evidence obtained support the hypothesis of the existence of heterogeneity of specification of labour input within the Swedish industrial sector. In other words, there is a number of industries for which it is not possible to employ the direct labour specification. The hypothesis that the labour input is an almost homogeneous function can be accepted in only two industries. Through the application of the Sato first stage we concluded that it is possible to reject in two industries the hypothesis that the elasticity of substitution between wage earners and

Table 4. *Summary of labour input estimation results*

ISIC	$L = f(WE, SE)$	
	Functional form of L	Elasticity of substitution between WE and SE
31	Cobb-Douglas	$\sigma = 1$
32	CES	$\sigma = 0.61672$
341	Cobb-Douglas	$\sigma = 1$
342	Almost homogeneous	$\sigma = 1.58773$, year 1963 1.58943, 1968 1.59071, 1974 1.59198, 1980
34	CES	$\sigma = 2.19584$
35	Linear	$\sigma = \infty$
36	Almost homogeneous	$\sigma = 1.02595$, 1963 1.02629, 1968 1.02691, 1974 1.02757, 1980
37	Linear	$\sigma = \infty$
38	Linear	$\sigma = \infty$

salaried employees is equal to one. In two cases the data examined emerge strong evidence of a simple Cobb-Douglas form for the labour put.

The results of this section are summarized in Table 4.

3.4 Model selection

The production function will be used a tool to permit us to explain in a compact form the most important characteristics of an industry. The nine industries will be divided into four categories of different labour specification: linear form, Cobb-Douglas form, CES form, and almost homogeneous form. Then we will proceed to estimate Cobb-Douglas production functions.

The Cobb-Douglas production function was used to obtain estimates of the output-capital and output-labour elasticities. Specifically, this means: (i) factor elasticities can be obtained directly as parameters of the first order, (ii) those elasticities are constant for any production level, (iii) it is not necessary to make assumptions regarding the market structure and (iv) it is more easy, compared with other production functions, to test alternative error specifications.

We use the following notation for the Cobb-Douglas function:

$$y = AK^{\alpha}L^{\beta} \quad (3.47)$$

where

y = output, K = capital and L = labour.

The labour input is given as

$$L = WE + SE \text{ (linear form)}$$

in the case of industries 35, 37 and 38; for the industries 31 and 341 the specification of labour input is given as a Cobb-Douglas form

$$L = (WE)^{a_1} (SE)^{a_2};$$

in the case of 32 and 34 the labour input is given as a CES function

$$L = [\delta(WE)^{-\rho} + (1-\delta)(SE)^{-\rho}]^{-1/\rho}$$

and finally in the case of industries 342 and 36 labour input is given as an almost homogeneous function

$$L = \delta(WE)^{1/k_1} + (1-\delta)(SE)^{1/k_2}$$

In most empirical studies involving the Cobb-Douglas function, one typically assume a multiplicative error term as reflected in the following equation:

$$y = AK^\alpha L^\beta e^u \quad (3.48)$$

where u is the error term which is assumed to be normally distributed, with mean zero and constant variance, i.e. $u \sim N(0, \sigma_u^2)$.

The equation (3.48) can be estimated by least squares after a logarithmic transformation. For this method to yield consistent estimates it is necessary that the inputs are distributed independently of u .

Also in the case of multiplicative specification Zellner, Kmenta and Dréze (ZKD) have shown that the reduced form equations for L and K are independent of the stochastic error term, if we assume that output is a random variable and that the decision unit maximizes expected profits.³⁶

Now, attempt is made to compare the above multiplicative error specification with the following additive error specification:

$$y = AK^\alpha L^\beta + v \quad (3.49)$$

where v is the error term which is assumed to be normally distributed with mean zero and constant variance, i.e. $v \sim N(0, \sigma_v^2)$.

From economic standpoint, an additive model (3.49) is consistent with the profit maximization assumption. The objective of this section is to

³⁶ See section 3.2.2.

determine whether the most appropriate error specification, in the case of Swedish manufacturing, is the multiplicative or the additive.

The next step is to choose between equations (3.48) and (3.49).³⁷

The likelihood functions for the two model specification (3.48) and (3.49) are given by

$$L_M = k \sigma_2^{-n} e^{-1/2n}$$

and

$$L_A = k \sigma_1^{-n} e^{-1/2n}$$

where L_A, L_M are the likelihood functions and σ_1, σ_2 are the standard errors, k =constant and n =number of observations.

Therefore, the likelihood-ratio statistic for this case is defined by

$$\lambda = \frac{L_A}{L_M} \prod_{i=1}^n y_i = \left(\frac{\sigma_2}{\sigma_1} \right)^n \prod_{i=1}^n y_i$$

or

$$\log \lambda = n (\log \sigma_2 - \log \sigma_1) + \sum_{i=1}^n \log y_i. \quad (3.50)$$

If $\log \lambda > 0$, then the data can be said to favour (3.49) and if $\log \lambda < 0$, then the log-linear model (3.48) is to be accepted.

3.5 Estimates and specification error tests

For computational purposes, we can rewrite the Cobb-Douglas form as

$$\frac{y}{L} = A(K/L)^\alpha L^h \cdot e^u \quad (3.51)$$

where $h = \alpha + \beta - 1$, the parameter of economies of scale.

The advantage of the equation (3.51) is that, in addition to obtaining the elasticity of output with respect to capital (α) we also obtain directly the economies of scale parameter (h). If h is not significantly different from zero, the constant returns to scale assumption need not be rejected. For a test of the hypothesis of constant returns to scale:

³⁷ For a full discussion of the problem of choosing between alternative models, see Atkinson (1970), Cox (1962), Mizon (1977) and Sargan (1969). The choice of the appropriate functional form is a crucial step in applied econometrics. Theil (1971) has pointed out the importance of discriminating between linear and log-linear models for forecasting.

Table 5. *Least squares estimates of the Cobb-Douglas function*

ISIC	$\log Y/L = \log A + \alpha \log K/L + h \log L + u$				
	$\log \hat{A}$	$\hat{\alpha}$	$\hat{h}$	R^2	D.M.
31	5.7232	.22369* (.04267)	0.00757 (0.27454)	.96	1.3
32	7.5153	-.4202 (.1956)	-0.84261 (0.7987)	.94	1.5
341	10.3009	.17089* (.06103)	-1.39673 (0.795813)	.61	1.3
342	8.7697	.31214* (.13153)	-1.55291 (0.94851)	.99	1.3
34	13.5276	.35972* (.14259)	-1.8966 (0.9887)	.96	1.8
35	8.894	.64332* (.09508)	-1.41849 (0.80864)	.87	1.9
36	4.4270	.60729* (.10654)	0.08801 (0.30756)	.99	1.8
37	3.10584	.52317* (.11003)	-0.24623 (0.41329)	.80	1.7
38	2.16076	.65817* (.1005)	0.11755 (0.23159)	.94	1.9

Note: Figures in brackets refer to the standard errors of the estimates.

* Denotes estimates significantly different from zero at the 0.05 level.

$$H_0: h = 0$$

we estimate the equation

$$\log (y/L) = \log A + \alpha \log \left(\frac{K}{L} \right) + h \log L + u. \quad (3.52)$$

That is, we performed ordinary least squares after correcting for serial correlation in the error terms (this section follows the more widely accepted practice of estimating equation (3.52) directly, with Cochrane-Orcutt technique) on a Cobb-Douglas production function of the form of (3.52) above.

As calculations were then carried out by estimating (3.52), we were able to test the hypothesis of constant returns to scale.

In examining the results of our calculation, see Table 5, we first discuss the coefficient h .

The variable L does not appear to be significant as a separate factor explaining variations in labour productivity. In all industries it was found that this coefficient, h , is statistically insignificant. Hence, the hypothesis of constant returns to scale can be accepted.

It is interesting to point out that the signs of the output elasticity with respect to capital, α , are in accordance with a priori expectations but their values vary among industries.

Since we have accepted the constant returns to scale hypothesis an additional estimation of output elasticities with respect to capital may be made by examining the following Cobb-Douglas production function

$$\frac{y}{L} = A \left(\frac{K}{L} \right)^{\alpha} \cdot e^u \quad (3.53)$$

with $\beta = 1 - \alpha$.

We have estimated the above equations including the time variable. Since the time variable has a coefficient which is statistically non-significant and the estimates of parameters obtained are not significantly different from zero or they have the wrong sign while the R^2 is high (an indication of multicollinearity), we did the regression without this variable. Thus, we have estimated the parameters without the variable t . The statistics R^2 and D.W. are not sensitive to this change in specification.

Table 6 sets out the results of estimating equation (3.53) by means of ordinary least squares. Parameter estimates are given with their standard error in parenthesis, together with the coefficient of determination R^2 , and the Durbin-Watson statistic D.W.

Looking at Table 6, we note the following:

- The signs are as expected. The interval of values of output elasticity of capital are (0.18779, 0.62866);
- the corresponding interval of values of output elasticity of labour are (0.37134, 0.81221). If ceteris paribus labour increases by 1 % the product will increase by about 0.8 % in the textile industry (32);
- the coefficients in all cases appear to be stable, i.e., $0 < \alpha < 1$ and $0 < \beta < 1$.
- The relatively large coefficients of capital (α) and small labour coefficients (β) probably reflect the treatment of the Cobb-Douglas function without the trend variable.
- As far as the use of unrestricted Cobb-Douglas function (3.52) or the restricted Cobb-Douglas function (3.53) is concerned one can learn from Tables 5 and 6 that in the case of most of the industries, an inclusion of the variable L (equation (3.52)) did not yield a higher coefficient of determination. These results empirically substantiate the constant returns to scale hypothesis, suggesting that the model (3.53) could be considered as yielding satisfactory results.

The Cobb-Douglas production function (3.53) was estimated by least squares to its log transformation under the assumption that it had a multiplicative stochastic term.

Table 6. *Least squares estimates of the Cobb-Douglas function (3.53)*

$\log Y/L = \log A + \alpha \log (K/L) + u$ Sample period 1963–1980					
ISIC	$\log \hat{A}$	$\hat{\alpha}$	$\hat{\beta} = -\hat{\alpha}$	R^2	D.W.
31	5.74598	.22257* (.01190)	.77743	.95	1.3
32	4.7331	.18779* (.0838)	.81221	.86	1.8
341	2.11688	.58851* (.12026)	.41149	.93	1.7
342	6.16249	.47901* (.07276)	.52099	.95	1.3
34	2.14578	.62715* (.07985)	.37285	.91	1.98
35	2.09066	.62866* (.08758)	.37134	.82	1.6
36	4.61011	.58289* (.06674)	.41711	.99	1.89
37	1.7879	.57170* (.04593)	.4283	.77	1.4
38	2.9961	.61974* (.04350)	.38026	.94	1.9

Note: Figures in brackets refer to the standard errors of the estimated.

* Denotes estimates significantly different from zero at the 0.05 level.

Now the same function but with different stochastic structure (additive) has been estimated directly. In other words, an attempt was made to estimate the following production function:

$$\frac{y}{L} = A \left(\frac{K}{L} \right)^{\alpha} + v. \quad (3.54)$$

Equation (3.54) is nonlinear and therefore requires a nonlinear estimating technique.

Hartley and Booker³⁸ demonstrate that if the independent variables are not subject to error and if the dependent variable is observed with an error which is normally distributed, then the nonlinear least squares iterative method has the following properties:

(i) the computational procedure is convergent for a finite n (number of observations)

³⁸ See Hartley and Booker (1965). For a detailed discussion, see Harvey (1981).

Table 7. *Non-linear estimates of the Cobb-Douglas function (3.49)*

$\frac{Y}{L} = A(K/L)^{\alpha} - v$ Sample period 1963-1980			
ISIC	Initial values	Final values	Remarks
31	$A = 1.6$ $\alpha = 0.4$	3.166 0.21838* (0.01214)	$\hat{\beta} = 0.78168$ D.W. = 1.4
32	$A = 1.6$ $\alpha = 0.4$	6.8092 1.1758* (0.3126)	D.W. = 0.59
341	$A = 1.6$ $\alpha = 0.4$	6.1199 0.50639* (0.0821)	$\hat{\beta} = 0.49361$ D.W. = 0.70
342	$A = 1.6$ $\alpha = 0.4$	4.602 0.49487* (0.05668)	$\hat{\beta} = 0.50513$ D.W. = 0.75
34	$A = 1.6$ $\alpha = 0.4$	11.554 0.56096* (0.05064)	$\hat{\beta} = 0.43904$ D.W. = 1.6
35	$\alpha = 1.6$ $\alpha = 0.4$	8.6309 0.80773* (0.08189)	$\hat{\beta} = 0.19227$ D.W. = 1.2
36	$A = 1.6$ $\alpha = 0.4$	10.967 0.56371* (0.071104)	$\hat{\beta} = 0.43629$ D.W. = 1.2
37	$A = 1.6$ $\alpha = 0.4$	6.1295 0.5646* (0.05649)	$\hat{\beta} = 0.4354$ D.W. = 1.3
38	$\alpha = 1.6$ $\alpha = 0.4$	20.74 0.59647* (0.02984)	$\hat{\beta} = 0.40353$ D.W. = 1.86

Note: Figures in brackets refer to the standard errors of the estimates.

* denotes estimates significantly different from zero at the 0.05 level.

(ii) the resulting estimators are asymptotically 100% as efficient as $n \rightarrow \infty$.

Various iterative methods are now available such as Gauss-Newton, modified Gauss-Newton, Newton-Raphson etc. In this paper a Gauss-Newton technique was used.

The estimation results for equation (3.54) are summarized in Table 7.

Looking at Table 7, we observe the following:

comparing the size of the elasticities with their corresponding standard errors, it is obvious that all elasticities are highly significant.

Judging the Durbin-Watson statistic, there is evidence of first order serial correlation.

In section 3.4 we raised the problem of comparing the results of linear estimation procedure (equation (3.53)) with those obtained by nonlinear estimation technique (equation (3.54)). The choice, as we have seen above, can be made according to the criterion $\log \lambda$. Based on calculated $\log \lambda$, the null hypothesis of the logarithm transformation of Cobb-Douglas was accepted in all cases. This also implies that the expected profit maximizing model was accepted in favour of profit maximizing.

The first and most obvious conclusion that can be drawn from the comparison is that both stochastic versions of the Cobb-Douglas model behaved similarly.

They both gave results that were broadly similar whichever stochastic error structure were assumed. This seems to indicate that starting from expected profit maximization rather than profit maximization is at least not inferior.

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Technological Progress and Factor Substitution in Swedish Manufacturing Industries

4.1 Introduction

The problem of technological progress was first introduced in studies of labour productivity. A number of studies¹ of productivity trends in the United States economy have been published, mainly since 1956.

These authors agree in attributing a predominant fraction of the increase in productivity per man hour to factors other than the increase in capital input. A pathbreaking study was published by Solow, "Technical Change and the Aggregate Production Function", in which he introduced technological progress into a production function. Using a Cobb-Douglas production function for United States non-farm output during 1909-1949, with a time trend to represent the increase in output for given inputs due to technological progress, Solow found that 87% of the increase in productivity per capita could be attributed to this factor.

Since the publication of Solow's paper, considerable attention has been devoted to the concept of technological progress, both in theoretical and empirical work. The nature of technological progress and the way in which it occurs have important implications for economic policy.

A distinction can be made between disembodied and embodied technological progress. Disembodied technological progress occurs as time passes. Without using a specific production function disembodied technological progress refers to any kind of shift in the production function of the firm. Thus, disembodied technological progress is measured by shifts in the production function.

It was "difficult" for economists to accept Solow's results, since they have long been acquainted with the theory that investment is one of the most important factors of economic growth. Thus, attention has been devoted to revise the production function in such a way so as to emphasize

¹ See Abramovitz (1956) and Aukrust (1959).

the key role that investment plays in economic growth. The crucial point is that all technological progress takes place through improvements that are embodied in new capital equipment. Thus far the main concern has been to establish the existence of a link between investment and technological progress and therefore the thesis of embodied technological progress emerged.

Embodied technological progress occurs as a result of improved quality of factor inputs and is measured by weighting capital and labour indices for quality changes.

Capital-embodied technological progress was estimated in Solow's² 1959 paper using vintage production functions. In order to find out how much investment is needed to increase the rate of output, the rates of both disembodied and embodied technological progress have to be estimated. Dale Jorgenson³ showed that a researcher can never distinguish a model of disembodied technological progress from a model of embodied technological progress on the basis of the models developed by Solow. In effect, he showed that the two concepts become econometrically indistinguishable when the rate of technological progress is regarded as being susceptible to unconstrained year to year variations.

Jorgenson points out that: "It has frequently been suggested that embodied and disembodied technical change are two different aspects of reality. For example, in commenting on Denison's study of total factor productivity (1962), based on disembodied technical change, Abramovitz says: 'The economic model which underlies Denison's calculations stands in sharp contrast to the model with which Robert Solow has been experimenting . . . The *factual gap* between the two views is profound and not really usefully attacked by speculation' . . ."⁴

After dropping the highly attractive assumption that technical change proceeds at constant exponential rates, we are able to show that one can never distinguish a model of embodied technical change from a model of disembodied technical change on the basis of factual evidence, such as that considered by Denison and Solow. Both types of technical change have precisely the same factual implications so that the 'factual gap' suggested by Abramovitz is entirely illusory."

Schultz and Denison⁵ among others appear to take the view that technological progress is partly the result of improvements in the quality of the labour input.

² See Solow (1960).

³ See Jorgenson (1966).

⁴ See Abramovitz (1962).

⁵ Proxy variable used to test the hypothesis that technical advance is partly the result of improvements in the quality of labour input, see T. W. Schultz and E. F. Denison.

Lundberg⁶ recognized that technological progress is partly the result of on-the-job learning obtained by human capital. Lundberg has named this phenomenon as the “Horndal effect”.

Another factor has been pointed out by Arrow:⁷ learning by doing. Noting the importance of the accumulation of experience in the development of new techniques, Arrow elaborated a vintage model where technological progress occurs only when new capital goods are actually built. In other words, Arrow proposed that the installation of capital equipment provides the key role for learning in the economy. Cumulative gross investment was therefore made the measure of accumulated production experience.

The point is that cumulative gross investment provides the vehicle of technological progress, as a continuing means of production and as such means of learning.

As Arrow points out, the higher the investment the greater will be the opportunities for learning the faster the rate of technological progress and thus the higher the level of production.

It is well known that the Hicksian neutral and non-neutral components of technological progress are indistinguishable in the context of the Cobb-Douglas production function which has been examined in the previous chapter. Also, we know that in the case of a Cobb-Douglas production function, the elasticity of substitution between capital and labour, σ , is unity. It follows, therefore, that any attempt at empirical estimation of two important parameters of production, i.e., the bias of technological progress and the elasticity of substitution, should proceed by using a more general production function than Cobb-Douglas.

This chapter thus uses the CES production function with labour input defined in Chapter 3, see Table 4.

Among the best known empirical studies at the manufacturing level for Sweden has been the one by Bergström.⁸ This chapter shall be regarded as a parallel to Bergström's study. However, the two studies differ with respect to the measurement of inputs. Bergström's study uses a direct (linear) labour input, without testing for the existence of such a specification.

As we have shown in chapter 3 we employ in most cases more complicated specifications for labour input (Cobb-Douglas, CES, and Almost homo-

⁶ See Lundberg (1961). Also Arrow (1962) has discussed this problem.

⁷ Arrow (1962) refers to the work of Wright (1936) on airframes, Verdoorn (1956) on national outputs, and Lundberg (1961) on the Horndahl iron works, all suggesting that increasing productivity is allied with experience.

For the learning by doing hypothesis, empirical evidence appears in Sheshinski (1967) and Panic (1976).

⁸ For detailed discussion, see Bergström (1982).

geneous). Hence in this chapter a nested or two stage production function is utilized.

We assume $y=f[g(WE, SE), K]$ where the capital input is separable from the labour input function g ; y is considered to be a CES function of g and K with constant elasticity of substitution, σ , and g to be an aggregate labour function of wage earners (WE) and salaried employees (SE) estimated in Chapter 3 (see Table 4). Therefore, we have to estimate the following production functions:

$$y = [a(WE^{\alpha_1}SE^{\alpha_2})^{-\rho} + (1-a)K^{-\rho}]^{-1/\rho} \quad (4.1)$$

in the case of the industries 31 and 341;

$$y = \{a[\beta WE^{-\mu} + (1-\beta)SE^{-\mu}]^{-\rho} + (1-a)K^{-\rho}\}^{-1/\rho} \quad (4.2)$$

in the case of the industries 32 and 34;

$$y = \{a[\delta WE^{1/k_1} + (1-\delta)SE^{1/k_2}]^{-\rho} + (1-a)K^{-\rho}\}^{-1/\rho} \quad (4.3)$$

in the case of the industries 342 and 36;

$$y = \{a(WE+SE)^{-\rho} + (1-a)K^{-\rho}\}^{-1/\rho} \quad (4.4)$$

in the case of the industries 35, 37 and 38.

The purpose of this chapter is to investigate if the above production function specifications are appropriate for Swedish manufacturing industries. To accomplish this we will test the conclusions of those earlier investigations to see if the new measure of labour input confirms that:

- (i) the time series estimates of the elasticity of substitution between capital and labour is found to be between zero and one and
- (ii) the technological progress is capital-using.

Consequently, this chapter presents the results of an empirical study of the Swedish manufacturing industries, undertaken as an application of the new labour input measure, and designed to throw light on some of the important questions about elasticity of substitution and the bias of technological progress.

This introductory background gives rise to the following organization for this chapter. Section 4.2 discusses the Resek⁹ test. This section also contains an application of Resek's test which illustrates that the assumption of Hicksian neutral technological progress is not consistent with Swedish manufacturing industries. Section 4.3 explores, both theoretically and empirically, the cost minimizing model. In subsection 4.3.1 we discuss the

⁹ See Resek (1963).

economic theory of the Diamond, McFadden “*Impossibility Theorem*”, in order to then specify the efficiencies of capital and labour in subsection 4.3.2. A dummy variable analysis is performed in the estimates of elasticity of substitution to test the stability of σ over time. Subsection 4.3.3 presents these results.

It is necessary to consider a possible simultaneous equations bias. In such a case the ordinary least squares estimator becomes highly suspect. In response to this problem the cost minimizing model has been estimated by two-stage least squares (2SLS). Section 4.4 present the production functions that were estimated by 2SLS.

Section 4.5 contains our empirical result on the joint modelling of labour and capital productivities. We there deal with a system of seemingly unrelated regression equations with one parameter restriction. Thus, in this section the emphasis will be on the information available for the estimates of elasticity of substitution and the capital and labour efficiencies from the joint estimation of productivity equations.

A dynamic version of the model can be obtained by introducing lagged variables. Applying the Koyck transformation leads to a very simple dynamic specification. Thus, the purpose of section 4.6 is to extend the static seemingly unrelated regression equations. Section 4.6 presents the dynamic model utilized here to (i) estimate the elasticity of substitution and (ii) the capital and labour efficiencies. Therefore, section 4.6 describes an alternative procedure and presents the resulting estimates.

4.2 Resek's test for neutrality

In many empirical papers on the estimation of production functions, the usual assumption is that technological progress is *Hicks neutral*. However, there is no reason a priori for this assumption and we will attempt in this section to provide some preliminary evidence that technological progress in the case of Swedish manufacturing industries during the period 1963–1980 was non-neutral in the Hicksian sense.

We begin with the linear neoclassical function which utilizes capital ($=K$) and labour ($=L$).

$$y = f(K, L, t)$$

where t represents the effect of time due to a technological progress and will be known as “technological progress”.

There are various ways of incorporating technological progress into a production function. This chapter considers a *linear homogeneous neoclassical* production function of *factor-augmenting* type:

$$y = f[\alpha(t) K, \beta(t) L] \quad (4.5)$$

Table 1. *Types of technological progress*

Types of technological progress by alternative invariant relationships	Relations between economic variables
Hicks-neutral	$\left(\frac{r}{w}, \frac{L}{K}\right)$
Harrod-neutral	$\left(r, \frac{Y}{K}\right)$
Solow-neutral	$\left(w, \frac{Y}{L}\right)$
Labour-combining	$\left(w, \frac{Y}{K}\right)$
Capital-combining	$\left(r, \frac{Y}{L}\right)$
Labour-decreasing	$\left(\frac{r}{w}, \frac{Y}{K}\right)$
Capital-decreasing	$\left(\frac{r}{w}, \frac{Y}{L}\right)$
Capital-additive	$\left(w, \frac{L}{K}\right)$
Labour-additive	$\left(r, \frac{K}{L}\right)$

where: r = return to capital, w = wage rate, Y = output, K = capital and L = labour.

where y is output at time t , K is capital, L is labour, $\alpha(t)$ and $\beta(t)$ are the efficiencies of K and L respectively.

Beckmann and Sato¹⁰ presented nine types of neutral¹¹ technological progresses which are generated from specific pairs of economic variables. Table 1 shows the pairs of variables and names of the associated neutralities.

Let us look briefly at the Hicks neutral case. In this case we may consider a simple linear function:

$$\frac{r}{w} = a_1 + b_1 \frac{L}{K} \quad (4.6)$$

and in order to test the uniqueness of the relationship between r/w and L/K

¹⁰ For detailed discussion, see Beckmann and Sato (1969).

¹¹ According to Beckmann and Sato neutrality means "the neutrality of the effects of technical change, that is to say, relationships between certain economic variables are invariant under technical change".

over time, we introduce the time variable in equation (4.6). Hence the equation (4.6) can be rewritten as

$$\frac{r}{w} = a_1 + b_1 \frac{L}{K} + c_1 t. \quad (4.7)$$

If c_1 is not significantly different from zero, it would appear that the relationship between r/w and L/K is unaffected by the passage of time. In this sense technological progress would be Hicks neutral.

In equation (4.5) technological progress is called *output-augmenting* if

$$f[\alpha(t) K, \beta(t) L] = h(t) g(K, L)$$

and technological progress is neutral in the Hicksian sense.

For example, the marginal rate of substitution is independent of technological progress $h(t)$ in the case of Hicks neutral technological progress.

In equation (4.5) technological progress is called *labour augmenting* or *Harrod neutral* if

$$f[\alpha(t) K, \beta(t) L] = f[K, \beta(t) L]. \quad (4.8)$$

In equation (4.5) technological progress is called *capital-augmenting* or *Solow neutral* if

$$f[\alpha(t) K, \beta(t) L] = f[\alpha(t) K, L]. \quad (4.9)$$

In 1962, Uzawa¹² showed that Hicks neutrality together with Harrod neutrality imply that the underlying production function is Cobb-Douglas. In other words, in the case of a Cobb-Douglas production function there is no way of distinguishing either Hicks neutral or Harrod neutral or from Solow neutrality.

Resek¹³ (see also Sato¹³) has proposed the following test for neutrality. Assume that the firm maximizes profit in competitive markets, then the first order conditions lead to:

$$f_K = \frac{\partial y}{\partial K} = r \quad (4.10)$$

$$f_L = \frac{\partial y}{\partial L} = w \quad (4.11)$$

¹² For the details, see Uzawa (1962).

¹³ See Resek (1963) and Sato (1970).

where r and w are capital rental and wage rates, respectively. The marginal rate of substitution of the two factors are defined by

$$m_t \equiv \frac{f_L(K, L, t)}{f_K(K, L, t)}. \quad (4.12)$$

Differentiating the logarithm of m_t totally with respect to time and using Euler's theorem, we obtain:

$$g_m = -f_{KL} \left(\frac{f}{f_K f_L} g_L - \frac{f}{f_K f_L} g_K \right) + \left(\frac{f_{Lt}}{f_L} - \frac{f_{Kt}}{f_K} \right) \quad (4.13)$$

where $g_x = \dot{x}/x = d/dt \log x$ and denotes the proportional rate of change over time.

By definition, σ , the elasticity of substitution, in the case of a linearly homogeneous production function is given by

$$\sigma = \frac{f_K f_L}{f f_{KL}}. \quad (4.14)$$

Then using σ , g_m can be written as:

$$g_m = \frac{1}{\sigma} (g_K - g_L) + \left(\frac{f_{Lt}}{f_L} - \frac{f_{Kt}}{f_K} \right). \quad (4.15)$$

As formulated by Hicks,¹⁴ the bias of technological progress is the difference between the proportional rates of growth of marginal products when $k=K/L$ is held constant:

$$B = \frac{f_{Kt}}{f_K} - \frac{f_{Lt}}{f_L}. \quad (4.16)$$

Technological progress is Hicks neutral, Hicks capital-using, and Hicks labour-using according to whether $B=0$, $B>0$, and $B<0$, respectively.

Combining g_m and B , we derive a relationship which incorporates the Hicksian index of bias, B :

$$g_m = \frac{1}{\sigma} (g_K - g_L) - B \quad (4.17)$$

or

$$\dot{k} = \sigma \cdot \dot{m}_t + \sigma B \quad \left(\text{since } m_t \equiv \frac{f_L}{f_K}, \text{ we have } \dot{m}_t = \dot{f}_L - \dot{f}_K \right).$$

¹⁴ See Hicks (1932).

Now, if we assume $\sigma = \text{constant}$ and $B = 0$ then from the last equation we obtain, given the assumption of competitive markets:

$$k = c \left(\frac{w}{r} \right)^\sigma, \quad c = \text{constant} > 0. \quad (4.18)$$

By drawing the scatter diagram of k vis-à-vis w/r we can test for neutrality. If we do not get a stable relation, then we reject the hypothesis that technological progress is Hicks neutral. As Takayama¹⁵ puts it "... if we cannot find such a stable relation, then we may conclude that this is due to a change in B , i.e., we reject the hypothesis that technological progress is Hicks neutral".

This is the method proposed by Resek to test for Hicks neutrality.

For example, Figures in Appendix might be interpreted to suggest that the functional relationship between the capital-labour ratio and the marginal rate of substitution— $K/L = h(w/r)$ —is unstable in the case of all industries.

Based on Resek's test we find that the available data are inconsistent with Hicks neutral technological progress in all industries.

This can be taken as evidence that nine industries are characterized by a non-neutral technological progress.

The findings of non-neutrality appear to be in agreement with earlier results obtained by Bentzel¹⁶ and Bergström.¹⁶

The criteria of Resek and the graphs do not explain the type of non-neutral technological progress. Therefore, other techniques must be sought.

4.3 The cost minimizing model

The estimation of the elasticity of substitution and the type of technological progress has been and continues to be an important topic in the econometrics of production. The production function may be shifting over time. This shift may be of the Hicks-neutral or of the factor augmenting type. In the previous chapter the Cobb-Douglas production function was estimated. In the case of Cobb-Douglas there are two fundamental problems: (a) the elasticity of substitution between capital and labour is unity and (b) it is not possible to distinguish between neutral and factor augmenting technological progress. Therefore these two problems are not independent of the specification of the production function.

Therefore, since our purpose in this chapter is to examine the nature of technological progress in the case of Swedish manufacturing industries, the Cobb-Douglas production function, for this purpose, was rejected since it can not distinguish certain types of technological progress.

¹⁵ See Takayama (1974).

¹⁶ See Bentzel (1978 and 1982), Bergström (1982).

The purposes of this section are (i) to briefly review the Diamond-McFadden¹⁷ methodology. Their methodology is used to identify empirically the exponential functions which describe technological progress in the context of a CES production function; (ii) to estimate the elasticity of substitution between capital and labour. These results are interpreted and some more general implications concerning the type of technological progress are drawn; and finally (iii) to examine the robustness of the results of our estimates of elasticity of substitution.

We begin with postulating the familiar CES¹⁸ production function:

$$y_t = \{[\alpha_t K_t]^{-\rho} + [\beta_t L_t]^{-\rho}\}^{-1/\rho} \quad (4.19)$$

where y_t is the output, K_t and L_t are measured capital and labour input, respectively, and α_t and β_t are efficiency indexes relating to K_t and L_t .

From (4.19) the marginal products of capital and labour are:

$$\begin{aligned} \frac{\partial y_t}{\partial K_t} &= F_K = \alpha_t^{-\rho} K_t^{-(1+\rho)} y_t^{1+\rho} \\ \frac{\partial y_t}{\partial L_t} &= F_L = \beta_t^{-\rho} L_t^{-(1+\rho)} y_t^{1+\rho}. \end{aligned} \quad (4.20)$$

Accordingly, the marginal rate of substitution of labour for capital, m_t , which is also the ratio of the marginal product of labour to that of capital is,

$$m_t \equiv (F_L/F_K)_t = \left(\frac{\beta}{\alpha}\right)_t^{-\rho} \left(\frac{K}{L}\right)_t^{1+\rho}. \quad (4.21)$$

Now, we assume that

- (i) firms minimize costs subject to the production function;
- (ii) the production function is given by (4.19);
- (iii) firms operate under competitive conditions in the factors markets.

By applying the above assumptions to (4.21) and equating the marginal rate of technical substitution to the ratio of factor prices we can derive the following cost minimization equation:

$$\left(\frac{\beta}{\alpha}\right)^{-\rho} \left(\frac{K}{L}\right)_t^{1+\rho} = \frac{w}{r} \quad (4.22)$$

where w/r is the factor-price ratio.

¹⁷ See Diamond, McFadden and Rodriguez in Fuss and McFadden (1978).

¹⁸ Constant-Elasticity-of-Substitution production function as first used by K. J. Arrow, H. B. Chenery, B. S. Minhas, and R. M. Solow (1961).

But, in equation (4.22) a specification has not been given concerning the form of factor augmentation.

4.3.1 The "Impossibility Theorem"

According to the well-known "Impossibility Theorem" in production theory¹⁹ we can determine the α_t and β_t . At present we shall briefly consider the implications of the "Impossibility Theorem" regarding the specifications of α_t and β_t .

From equation (4.17) we observe that there are two unknowns (σ, B), thus there exists an infinite number of solutions for the two parameters that will satisfy this equation. In economic terms, this implies that neither the bias nor the elasticity of substitution is identified. This is the famous "Impossibility Theorem" stemming from the work of Diamond and McFadden.¹⁹ The general message of this theorem is that unless either the value of elasticity of substitution or the bias of technological progress is known a priori, it is not possible to estimate the production function and a general form of technological progress. Since there is no way of knowing the bias of technological progress, the assumption that all technological progress is factor augmenting or non-factor augmenting is essential to its identification.

Further, from the "Impossibility Theorem" it follows that one *cannot make* the assumption that technological progress is *both neutral and factor augmenting at the same time*. As the "Impossibility Theorem" implies, the time path of technological progress *must be smooth* in order for the production function to be identified.

This is the method *used to obtain identification* in this section. Exponential functions that are linear ($e^{\lambda t}$), quadratic ($e^{\lambda t^2}$), and power functions (t^{λ}) were tried in this section.

We will now specify the ways that α_t and β_t are represented by t . The most simple way is to specify α_t and β_t as

$$\alpha_t = \beta_t = e^{\lambda t}. \quad (4.23)$$

This specification implies that technological progress proceeds at an equally proportional rate over time. These specifications have been extensively used.

A more general representation²⁰ is given as

$$\alpha_t = \alpha_0 e^{\lambda_1 t + \lambda_L t^2} \quad \beta_t = \beta_0 e^{\lambda_2 t + \lambda_K t^2}. \quad (4.24)$$

¹⁹ See Diamond and McFadden (1965) and Diamond, McFadden and Rodriguez in Fuss and McFadden (1978).

²⁰ See Levhari and Sheshinski (1973).

By directly comparing (4.24) and (4.23) we see the advantage of (4.24) with respect to (4.23), as the latter allows for all kinds of monotonic technological progress. The convenience of the (4.23) is self-revealing once we realize that our estimated equation is given in logarithms—see below equations (CR), (DR), and (IR). The form (4.23) had been extensively used in the empirical analysis of production. However, Moroney²¹ has found that the correlation between the time variable and the logarithm of wage rate is positive and very high.

This implies that we encounter the problem of multicollinearity. It consequently becomes difficult to disentangle the separate influences of the explanatory variables and to obtain precise estimates of their individual influence.

But if multicollinearity is not a problem, then it is preferable to avoid the use of (4.23) in favour of (4.24).

Obviously, if there is a problem of multicollinearity in t and $\log w$ then the specification (4.24) cannot give better results.

One way to reduce the degree of multicollinearity is to consider the case

$$\alpha_t = \alpha_0 e^{\lambda_L t^2}, \quad \beta_t = \beta_0 e^{\lambda_K t^2}. \quad (4.25)$$

This specification implies that technological progress proceeds at a non-constant increasing rate over the relevant period. The use of equation (4.25) is found in the works of Weitzman and Toda.²²

This specification implies that technological progress proceeds at a non-constant increasing rate over the relevant period.

Another specification of α_t (and β_t) which can be used in order to reduce the degree of multicollinearity is

$$\alpha_t = \alpha_0 t^{\lambda_L}, \quad \beta_t = \beta_0 t^{\lambda_K}, \quad (4.26)$$

This specification implies that technological progress is increasing at a decreasing rate.²³

If we combine the above three specifications of technological progress with equation (4.22) we obtain the following cases:

(1) a specification where the differential rate ($\lambda_L - \lambda_K$) is assumed to proceed at a constant rate

$$\log \left(\frac{K}{L} \right)_t = a_0 + a_1 \log \left(\frac{w}{r} \right) + a_2 t + u \quad (\text{CR})$$

²¹ See Moroney (1966).

²² See Weitzman (1970) and Toda (1979).

²³ For empirical results, see Fishelson (1974), Lianos (1971) and Moroney (1966).

(2) a specification where it is assumed to increase at decreasing rate

$$\log \left(\frac{K}{L} \right)_t = a_0 + a_1 \log \left(\frac{w}{r} \right) + a_2 \log t + u \quad (\text{DR})$$

and

(3) a specification where it is assumed to increase at an increasing rate

$$\log \left(\frac{K}{L} \right) = a_0 + a_1 \log \left(\frac{w}{r} \right) + a_2 t^2 + u \quad (\text{IR})$$

where

$$a_0 = (1 - \sigma) \log \left(\frac{\beta_0}{\alpha_0} \right)$$

$$a_1 = \sigma$$

$$a_2 = (1 - \sigma) (\lambda_L - \lambda_K) \Rightarrow \lambda_L - \lambda_K = \frac{a_2}{1 - \sigma}$$

$$\text{and } u \sim N(0, \sigma_u^2).$$

In our process of choice the consistency of the data specification of α_t and β_t , we adopted a procedure which combine theoretical considerations as well as empirical observations, designed to extract the maximum of information from our time series data.

We specify two criteria according to which the selection between (CR), (DR) and (IR) will be made: Criterion 1, with regards to economic criterion involves a specification for which the estimates are not in contradiction with a priori economic expectations. For example, the elasticities of substitution is expected to be positive. As regards the magnitude of the elasticities, we expect none of the elasticities of substitution to be statistically insignificant different from zero. We expect the elasticity of substitution to be different from zero: (i) because we assume the existence of a neoclassical production function which is a continuous and therefore we exclude the Leontief production function; (ii) a second reason derives from the empirical fact that the estimation of the elasticity of substitution in the case of Swedish manufacturing are less than one and greater than zero (see Bergström). Criterion 2, necessitates that the statistics selected to test for the correct specifications are (a) R^2 's, (b) the standard error of each independent variable and, (c) the Durbin-Watson statistics.

The results of estimating equations (CR), (DR) and (IR) during the period in question are given in Tables 2, 3 and 4. The examination of Tables 2, 3 and 4 reveals the following main points:

Table 2. *Ordinary least squares estimates of*

$$\log\left(\frac{K}{L}\right) = \alpha_0 + \alpha_1 \log\left(\frac{w}{r}\right) + \alpha_2 t + u \quad (\text{CR}).$$

Sample period 1963–1980

ISIC	Estimated coefficients				Summary statistics	
	$\hat{\alpha}_0$	$\hat{\alpha}_1 = \hat{\sigma}$	$\hat{\alpha}_2$	$\widehat{\lambda_L - \lambda_K}$	R^2	D.W.
31	5.8964	-.01438 (.03498)	.05332* (.00225)	N.C.	.967	0.65
32	4.77661	-.037002 (.06566)	.06794* (.00649)	N.C.	.971	1.1
341	4.7236	.03656 (.05742)	.07738* (.00445)	N.C.	.978	0.6
342	3.36037	.016989 (.05713)	.06645* (.00458)	N.C.	.976	0.4
34	4.61836	.17935* (.05007)	.07587* (.00391)	.09245	.986	0.6
35	4.0307	.03029 (.04384)	.06080* (.00450)	N.C.	.984	1.3
36	3.32771	.12459 (.07285)	.09070* (.00652)	N.C.	.970	1.8
37	4.0011	.14565 (.07321)	.06629* (.00289)	N.C.	.994	1.4
38	1.95008	.12908 (.11375)	.07788* (.00956)	N.C.	.959	0.9

Note: Number in parentheses are the estimated standard errors of the respective coefficients.

* Denotes estimates significantly different from zero at the 0.05 level.

N.C. = Not computed since σ is not significantly different from zero.

- The R^2 is not significantly different for the three models. Thus, using the goodness of fit alone, it would be inappropriate to prefer one equation over the other.
- The problem with the specification (CR) is the large number of not significant estimates of the elasticity of substitution. More precisely, in eight out of nine cases the elasticity of substitution turned out to be insignificantly different from zero. One possible explanation for these results is the existence of the problem of multicollinearity. In almost all industries the correlation coefficient between the $\log(w/r)$ and t was in the neighborhood of 0.8. This implies that the specification of (CR) is subject to multicollinearity. In other words it becomes very difficult to study the separate influences of the explanatory variables. Consequent-

Table 3. *Ordinary least squares estimates of*

$$\log \left(\frac{K}{L} \right) = \alpha_0 + \alpha_1 \log \left(\frac{w}{r} \right) + \alpha_2 \log t + u \quad (\text{DR}).$$

Sample period 1963–1980

ISIC	Estimated coefficients				Summary statistics	
	$\hat{\alpha}_0$	$\hat{\alpha}_1 = \hat{\sigma}$	$\hat{\alpha}_2$	$\widehat{\lambda_L - \lambda_K}$	R^2	D.W.
31	4.84195	.20109* (.0915)	.26949* (.04346)	.33732	.887	0.63
32	3.17453	.24905* (.11846)	.25772* (.07797)	.34319	.861	0.8
341	3.09086	.28493* (.10862)	.39695* (.05599)	.55512	.922	1.4
342	2.26176	.18709* (.07576)	.36438* (.04046)	.44824	.944	1.2
34	3.0375	.25829* (.10109)	.39518* (.05261)	.53279	.927	1.1
35	2.96459	.22435* (.11335)	.26016* (.07703)	.33541	.877	0.8
36	1.41464	.40478* (.15711)	.38352* (.09228)	.64433	.911	1.4
37	3.0154	.22464* (.07003)	.29054* (.05769)	.37472	.926	1.2
38	0.59887	.36623* (.13675)	.38984* (.07654)	.61511	.920	1.2

Note: Number in parentheses are the estimated standard errors of the respective coefficients.

* Denotes estimates significantly different from zero at the 0.05 level.

ly, it is difficult to reject the null hypothesis that the elasticity of substitution is different from zero.

- Since too many estimates of the elasticity of substitution derived from (CR), seem to contradict our a priori expectations, then this is an indication that the estimated equation (CR) is probably misspecified.

Consequently, in order to reconcile the large number of insignificantly elasticities of substitution it was concluded that technological progress had not proceeded at a constant rate.

- It is interesting to note that with the specification (DR) we have significantly different from zero point estimates of the elasticity of substitution. Also with this specification, and most important, the estimated rates of technological progress are unreasonably high in all cases. One possible explanation for the high rates obtained is that the specification (4.26) is

Table 4. *Ordinary least squares estimates of*

$$\log\left(\frac{K}{L}\right) = \alpha_0 + \alpha_1 \log\left(\frac{w}{r}\right) + \alpha_2 t^2 + u \quad (\text{IR}).$$

Sample period 1963–1980

ISIC	Estimated coefficients				Summary statistics	
	$\hat{\alpha}_0$	$\hat{\alpha}_1 = \hat{\sigma}$	$\hat{\alpha}_2$	$\widehat{\lambda_L - \lambda_K}$	R^2	D.W.
31	1.5905	.16997* (.08452)	.06872* (.0035)	.08267* (.00481)	.947	1.4
32	4.2426	.23366* (.09719)	.00282* (.00036)	.00368* (.00057)	.952	1.1
341	4.3155	.15449* (.06332)	.00341* (.00052)	.00403* (.00077)	.911	1.2
342	2.9718	.13499* (.05741)	.00283* (.00051)	.00327* (.00072)	.881	1.3
34	4.21996	.13920* (.0572)	.00336* (.00051)	.00390* (.00066)	.958	1.2
35	3.3447	.20313* (.05697)	.00229* (.00029)	.00287* (.00037)	.956	1.4
36	2.0919	.35179* (.12217)	.00327* (.00055)	.00504* (.00084)	.943	1.9
37	3.63789	.16332* (.06108)	.00262* (.00038)	.00313* (.00052)	.951	1.6
38	0.99181	.37657* (.15856)	.00289* (.00068)	.00464* (.00089)	.901	1.7

Note: Number in parentheses are the estimated standard errors of the respective coefficients.

* Denotes estimates significantly different from zero at the 0.05 level.

not suitable for such estimation. Therefore, the model (DR) was rejected on the grounds that it was misspecified.

- Our inability to obtain statistically significant estimates for the values of σ deprives us with the possibility of using the specifications (CR) and (DR). Thus, to the extent that these tests are conclusive, we can eliminate specifications (CR) and (DR).
- Our inability to obtain statistically significant results and consistent with a priori expectations raised the obvious question of the applicability of the (IR) model to the Swedish experience.

The estimated elasticity of substitution is both significantly different from zero and significantly less than unity.

- In summary, it appears that the specification (IR) has removed possible problems associated with multicollinearity.

- Our empirical results, see Tables 2, 3 and 4, indicates that the estimates of the elasticity of substitution are sensitive to the specification of technological progress. To get an idea of the sensitivity of the estimates, notice that the elasticity of substitution in the industry 38 is estimated to be 0.12908 and statistically insignificant for the trended function (CR), 0.36623 for the (DR) specification and 0.37657 for the (IR) specification.
- Therefore, the model with technological progress which the differential rate, $\lambda_L - \lambda_K$, of technological progress increases at an increasing rate was accepted in the case of Swedish manufacturing industries. We will argue that particularly in a forecasting context the model (IR) can be an extremely serious problem. The forthcoming analysis is based on model (IR). In the next section we present the bias of technological progress.

4.3.2. *Type of factor augmentation*

The specific assumption with regard to the augmentation of factors is that:

$$\alpha_t = \alpha_0 e^{\lambda_K t^2}, \beta_t = \beta_0 e^{\lambda_L t^2}, \lambda_K, \lambda_L > 0. \quad (4.27)$$

We are now able to define the factor augmenting biases in technological progress. According to Hicks, inventions are to be designated as neutral, capital-using, or labour-using, according to whether they have caused the marginal productivity of labour relative to the marginal productivity of capital to remain unchanged, to decrease or increase respectively, for a given capital-labour ratio.

Following conventional assumptions and procedures, the bias (B) of technological progress and the rate of change of the share of labour ($\dot{s}/s$) in value-added are given by the following equations:

$$B = \frac{F_{Kt}}{F_K} - \frac{F_{Lt}}{F_L} \quad (4.28)$$

$$\left(\frac{\dot{s}}{s}\right) = -(1-s) \left[B + \frac{\sigma-1}{\sigma} \left(\frac{\dot{k}}{k}\right) \right] \quad (4.29)$$

where F_{Kt} and F_{Lt} are the changes over time of the marginal products of capital and labour respectively; where $(\dot{s}/s)$ is the rate of change of the share of labour in value added; where s is labour share; σ is the elasticity of substitution between capital and labour; $(\dot{k}/k)$ is the rate of change in the capital-labour ratio.

Using equations (4.22) and (4.27) in equations (4.28) and (4.29), one obtains:

$$B = -2 \left(\frac{\sigma-1}{\sigma} \right) (\lambda_L - \lambda_K) t = - \left(\frac{\sigma-1}{\sigma} \right) (\lambda_L - \lambda_K) t' \quad (4.30)$$

$$\left(\frac{\dot{s}}{s} \right) = -(1-s) \left(\frac{\sigma-1}{\sigma} \right) \left[\left(\frac{\dot{k}}{k} \right) - (\lambda_L - \lambda_K) t' \right] \quad (4.31)$$

where $t=1,2,3,\dots,18$ and t' is the geometric mean of $2t$. ($t' = (2(1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \dots))^{1/18} = 7.4$ —see Fishelson (1974).

It is clear from equation (4.30) that the case of Hicksian neutrality can be expressed in this model by the equality of growth of capital and labour augmentation, that is, when $\lambda_K = \lambda_L$ regardless of the value of the elasticity of substitution.

We can also see that what we consider a labour augmenting bias, can be defined as:

$$\begin{cases} 0 < \sigma < 1 & \text{and } \lambda_L > \lambda_K & \text{(capital-using)} \\ \sigma > 1 & \text{and } \lambda_L > \lambda_K & \text{(labour-using)} \end{cases} \quad (B1)$$

In the other two combinations technological progress has a capital augmenting bias, that is, when

$$\begin{cases} 0 < \sigma < 1 & \text{and } \lambda_K > \lambda_L & \text{(labour-using)} \\ \sigma > 1 & \text{and } \lambda_K > \lambda_L & \text{(capital-using)} \end{cases} \quad (B2)$$

We shall make use of (B1) and (B2) in order to interpret the nature of factor augmentation in the Hicksian sense of factor-using biases in technological progress.

The following equation

$$\log \left(\frac{K}{L} \right) = a_0 + a_1 \log \left(\frac{w}{r} \right) + a_2 t^2 + u \quad (4.32)$$

where

$$a_0 = (1-\sigma) \log \left(\frac{\beta_0}{\alpha_0} \right) \quad (4.33)$$

$$a_1 = \sigma \quad (4.34)$$

$$a_2 = (1-\sigma) (\lambda_L - \lambda_K) \quad (4.35)$$

provides us with a simple least squares regression model needed to estimate both the elasticity of substitution and the bias in technological change in terms of the differences in the exponential growth rates of efficiency of the two factors.

The restrictive nature of this model is that (4.32) cannot be used to distinguish between neutral technological progress ($\lambda_L - \lambda_K = 0$, $\lambda_L, \lambda_K > 0$) and zero technological progress ($\lambda_L = \lambda_K = 0$). But, according to Moroney "... it can be used to identify differential rates of disembodied technological progress".

It can be seen from Table 4 that the coefficients of the time variable in 9 of the industries are significantly different from zero. The estimates of differential rate of factor augmentation, $\hat{\lambda}_L - \hat{\lambda}_K$, are given by the ratio $\hat{a}_2/(1-\hat{\sigma})$. In this case the approximate standard error is obtained by the Taylor expansion given that the function converge:

$$S_{(\lambda_L - \lambda_K)} = \left\{ \left(\frac{\hat{a}_2}{(1-\hat{\sigma})^2} \right)^2 \text{var}(\hat{a}_1) + \left(\frac{1}{1-\hat{\sigma}} \right)^2 \text{var}(\hat{a}_2) + 2 \frac{\hat{a}_2}{(1-\hat{\sigma})^3} \text{cov}(\hat{a}_1, \hat{a}_2) \right\}^{1/2}. \quad (4.36)$$

In all case the estimates of $(\lambda_L - \lambda_K)$ turned out to be positive (see Table 4). This means that there was a labour augmenting bias in technological progress in these industries. Combining these results with the former conclusion that $\hat{\sigma}$ is less than unity, implies that there was a capital-using bias in technological progress for these industries.

4.3.3 A test for the stability of σ

We shall now test the stability of constancy of σ . Since σ may change over time, the stability of the σ 's obtained by the equation (4.32) is of vital importance. To test the stability of the σ 's over time a dummy variable approach was used.²⁴ We have evaluated the structural stability of equation (4.32) over the period 1963–1980 by dividing the time period into two subperiods (1963–1971) and (1972–1980). The choice of the subperiods has been dictated by the degrees of freedom.

Assume that $D=1$ for observations lying in the first subperiod (1963–1971) and $D=0$ for observations lying in the second subperiod.

In this case we can write our equation (4.32) as:

$$\log \left(\frac{K}{L} \right)_t = a_0 + a_1 \log \left(\frac{w}{r} \right)_t + a_2 t^2 + a_3 \left(D \cdot \log \left(\frac{w}{r} \right)_t \right) + u_t. \quad (4.37)$$

If a_3 is statistically insignificant, then a_1 gives the estimate of σ which is common in both subperiods. If a_3 is statistically significantly different from zero, then the estimate of σ of the first subperiod is $a_1 + a_3$, where a_1 is the estimate of σ of the second subperiod. Thus, with the help of the dummy approach, we can test the hypothesis:

²⁴ See Gujurati (1970).

Table 6. *Test results for structural shift equation estimated*

$$\log\left(\frac{K}{L}\right) = \alpha_0 + \alpha_1 \log\left(\frac{w}{r}\right) + \alpha_2 t^2 + \alpha_3 D \cdot \log\left(\frac{w}{r}\right) + u$$

$$D = \begin{cases} 1, & 1963-1971 \\ 0, & 1972-1980 \end{cases}$$

ISIC	Estimated coefficients				Summary statistics	
	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	R^2	D.W.
31	5.7620	.10545* (.04112)	.00186* (.00037)	.02662 (.01625)	.943	1.1
32	4.26426	.10319* (.0384)	.00274* (.00046)	-.00334 (.01241)	.953	1.4
341	4.37416	.16520* (.0525)	.00285* (.00072)	-.00211 (.01918)	.918	1.3
342	3.0065	.15112* (.06253)	.00226* (.00072)	-.02244 (.02045)	.890	1.5
34	4.26272	.15376* (.05597)	.00276* (.00071)	-.02154 (.01789)	.919	1.2
35	3.4594	.19402* (.05722)	.00205* (.00036)	-.01278 (.01169)	.959	1.5
36	2.20161	.34366* (.12513)	.00301* (.00069)	-.01418 (.02168)	.954	1.6
37	3.72131	.16026* (.06135)	.00236* (.00048)	.01369 (.01495)	.954	1.6
38	1.03137	.38010* (.16329)	.00264* (.00091)	-.01134 (.02657)	.902	1.7

Note: Number in parentheses are the estimated standard errors of the respective coefficients.

* Denotes estimates significantly from zero at the 0.05 level.

$H_0: \sigma_1 = \sigma_2$ (where σ_1 and σ_2 are the elasticities of substitution for the first and second subperiod, respectively).

If α_3 is statistically insignificant then the H_0 cannot be rejected. From which we can conclude that there is no evidence of change in the value of σ from the first subperiod to the second subperiod.

To see what actually happened, consider the picture given in Table 6.

As this table shows, none of the α_3 's were statistically different from zero, therefore, we can say that $\sigma_1 = \sigma_2$. Hence, our null hypothesis was not rejected and consequently we conclude that the σ 's estimated from the cost minimizing model have not changed during the period being studied.

4.4 OLS vs. 2SLS

It is necessary to consider a possible simultaneous equations bias which may effect the ordinary least squares estimates of equation (4.32). The application of OLS in this case may yield biased and inconsistent estimates. Since this could critically affect the interpretation of the estimates of equation (4.32) two-stage least squares (2SLS) estimates of this equation have instead been obtained in this section.

In their paper, Moroney and Ferguson²⁵ have reestimated the constrained cost minimization equation (4.32) for each industry by applying the two stage least squares (2SLS) technique.

The reason for their use of this technique is that they did not test the assumption that "the disturbances in the i -th industry are independent of those in the j -th industry for all $i \neq j$ ".²⁵

That is to say, the disturbance in equation (4.32) of i -th industry has a distribution depending upon the values of the disturbances in the other industries.

The most important conclusion of their paper is that "the 2SLS estimates yield relatively more efficient estimates of the regression coefficients for U.S. manufacturing industries".²⁵

In what follows, we will use the Moroney-Ferguson framework on time series data for Swedish manufacturing industries (1963-1980).

Hence, in order to take into account the correlation of the disturbances across industries, 2SLS estimates²⁶ were computed for equation (4.32). Thus, equation (4.32) was then re-estimated by applying the two-stage least-squares technique, and the results are reported in Table 7.

Having completed the estimation, the next step is to examine the superiority, in terms of efficiency, of the 2SLS estimates.

In practice, the researcher wants to select an estimation for σ (and for $\lambda_L - \lambda_K$) with the smallest bias and mean square error.

Since bias is a part of means square error, it is reasonable to consider only the variance or the t -statistics.

The relative efficiency of $\hat{\sigma}_{OLS}$ (or $\lambda_L - \lambda_K$) to the estimator $\hat{\sigma}_{2SLS}$ is defined to be

$$REF = \frac{\text{Variance of OLS estimator}}{\text{Variance of 2SLS estimator}}, \quad (4.38)$$

i.e., the ratio of the variance of the OLS estimator to the variance of 2SLS. In the case where REF is less than unity, OLS, is relatively more efficient than the 2SLS.

²⁵ See Moroney and Ferguson (1970).

²⁶ In 2SLS estimates, the instruments were P (population), C (constant), T (time), TT (squared trend).

Table 7. 2SLS estimates of equation

$$\log\left(\frac{K}{L}\right) = \alpha_0 + \alpha_1 \log\left(\frac{w}{r}\right) + \alpha_2 t^2 + u. \text{ Sample period 1963-1980}$$

ISIC	Estimated coefficients			Summary statistics D.W.
	$\hat{\alpha}_0$	$\hat{\alpha}_1 = \hat{\sigma}$	$\hat{\alpha}_2$	
31	2.36338	.18807* (.08468)	.00067* (.00009)	1.7
32	6.3585	.27886* (.12989)	.00447 (.00629)	0.8
341	15.885	.24857* (.11958)	.00958 (.00778)	1.4
342	5.30477	.21309* (.10963)	.00346* (.00061)	1.7
34	9.6167	.17135* (.08241)	.00924 (.00724)	1.3
35	2.2495	.41103* (.12429)	.00142* (.0005)	1.9
36	-3.4006	.29231* (.13256)	.00097 (.0035)	1.7
37	2.56045	.37989* (.15299)	.00147 (.00087)	1.7
38	-2.25667	.38639* (.16552)	.00884 (.06648)	1.9

Note: Number in parentheses are the estimated standard errors of the respective coefficients.

* Denotes estimates significantly different from zero at the 0.05 level.

From Tables 4 and 7 we observe that in all nine industries the 2SLS estimates are inferior to the OLS estimates.

From Tables 4 and 7 we observe that the point 2SLS estimates of elasticity of substitution were higher than those for the OLS estimates, i.e.,

$$\hat{\sigma}_{2SLS} > \hat{\sigma}_{OLS}.$$

Berndt,²⁷ for example, finds that "the 2SLS estimates of σ are larger than OLS estimates".

The results given in Table 7 support those given in Table 4, which showed that $0 < \sigma < 1$ in all nine industries. It is interesting to observe that the 2SLS estimates of $\lambda_L - \lambda_K$ have higher standard errors than the OLS estimates in all nine industries. Thus, the OLS estimates of $\lambda_L - \lambda_K$ are relatively more efficient than the 2SLS estimates.

²⁷ See Berndt (1976).

In comparing our findings with those of Moroney and Ferguson for the U.S. manufacturing industries, we observe that while Moroney and Ferguson found 2SLS more efficient than OLS in all manufacturing industries, for σ and $\lambda_L - \lambda_K$, this study found OLS more efficient than 2SLS.

In summary, OLS seems to perform better than 2SLS, in terms of standard errors. As a result, the OLS estimates were accepted for this study.

4.5 Bergström–Melander–Kalt model

The main problem with the cost minimizing model is that labour and capital augmenting coefficients cannot be identified separately.

Our interest in this section is to derive simultaneously estimates of σ , λ_L and λ_K .

Several researchers have estimated $\lambda_L - \lambda_K$ or λ_L with $\lambda_K = 0$ while less attention has been given to methods of estimating both λ_L and λ_K .

The works of Kalt,²⁸ Bergström and Melander²⁹ are only a few in the latter category. Kalt has estimated λ_L and λ_K utilizing capital and labour productivities. Interestingly enough, Kalt states that:

... the role of technological change in the U.S. aggregate production function, (CES), imply annual rates of labor and capital augmenting technological change in the U.S. private domestic economy of 2.2% and -0.69% respectively, over the 1929-67 sample period. The estimate of the rate of labour augmentation is virtually identical to the least squares estimate and is significantly different from zero at the 1% level. The rate of capital augmentation, however, now appears to have been negative over the sample period ...

Bergström and Melander have clarified the restrictions of using the equilibrium equality between marginal productivities and factor prices. Bergström and Melander used a full information maximum likelihood method and they found in the case of Swedish industry that:

... most estimates indicate an elasticity of substitution of less than one. Technological development has been labor saving. The adjustment of labour input towards optimal levels was much faster than that of capital.

This section builds upon Bergström–Melander–Kalt's framework and presents some further evidence on the degree of substitution between capital and labour and the estimates of capital and labour augmentation coefficients.

To begin with, we make the following assumptions:

²⁸ See Kalt (1978).

²⁹ See Bergström (1982) and Bergström and Melander (1982).

(i) We assume that firm's production function can be expressed as a CES function, i.e.,

$$Y_t = \left[(\alpha_0 e^{\lambda_K t^2} K_t)^{-p} + (\beta_0 e^{\lambda_L t^2} L_t)^{-p} \right]^{-1/p}. \quad (4.39)$$

(ii) The firm's decision problem can be stated as $\max \pi$, where π =profits.

From the first order conditions which result from this maximization problem, the following familiar equations can be obtained:

$$\frac{\partial y}{\partial L} = (\beta_0 e^{\lambda_L t^2})^{-p} \left(\frac{y}{L} \right)^{1+p}$$

$$\frac{\partial y}{\partial K} = (\alpha_0 e^{\lambda_K t^2})^{-p} \left(\frac{y}{K} \right)^{1+p}.$$

Assuming competitive conditions we obtain:

$$(\beta_0 e^{\lambda_L t^2})^{-p} \left(\frac{y}{L} \right)^{1+p} = w \quad (4.40)$$

$$(\alpha_0 e^{\lambda_K t^2})^{-p} \left(\frac{y}{K} \right)^{1+p} = r \quad (4.41)$$

where w =wage rate

r =user cost of capital.

Taking logarithms, (4.40) and (4.41) become:

$$\left. \begin{aligned} \log \left(\frac{y}{L} \right) &= \frac{p}{1+p} \log \beta_0 + \frac{1}{1+p} \log w + \frac{p \lambda_L}{1+p} t^2 \\ \log \left(\frac{y}{K} \right) &= \frac{p}{1+p} \log \alpha_0 + \frac{1}{1+p} \log r + \frac{p \lambda_K}{1+p} t^2 \end{aligned} \right\} \quad (4.42)$$

The statistical counterpart of (4.42) is

$$\left. \begin{aligned} \log \left(\frac{y}{L} \right) &= \alpha_0 + \alpha_1 \log w + \alpha_2 t^2 + u_L \\ \log \left(\frac{y}{K} \right) &= \beta_0 + \beta_1 \log r + \beta_2 t^2 + u_K \end{aligned} \right\} \quad (4.43)$$

where

$$\hat{\sigma} = \hat{\alpha}_1 = \beta_1$$

$$\hat{\lambda}_L = \frac{\hat{\alpha}_2}{1 - \hat{\alpha}_1}$$

Table 8. *Seemingly unrelated estimates of*

$$\log\left(\frac{Y}{L}\right) = \alpha_0 + \alpha_1 \log w + \alpha_2 t^2 + u_L$$

$$\log\left(\frac{Y}{K}\right) = \beta_0 + \beta_1 \log r + \beta_2 t^2 + u_K.$$

Sample period 1963–1980

ISIC	Estimated coefficients						
	$\hat{\alpha}_0$	$\hat{\alpha}_1 = \hat{\beta}_1 = \hat{\sigma}$	$\hat{\alpha}_2$	$\hat{\lambda}_L$	$\hat{\beta}_0$	$\hat{\beta}_2$	$\hat{\lambda}_K$
31	6.14146	.19058* (.04061)	.0071* (.001)	.0088* (.0028)	0.28552	-.0018* (.00012)	-.00222* (.00024)
32	3.67815	.27208* (.06512)	.0141* (.0026)	.0194* (.0033)	0.36673	-.0067* (.00019)	-.0092* (.00026)
341	3.63407	.22453* (.05876)	.0152* (.0041)	.01960* (.0061)	-0.26599	-.00167* (.00017)	-.00215* (.00025)
342	3.07792	.20357* (.10149)	.0186* (.003)	.0234* (.009)	0.00234	-.00074* (.00028)	-.00093* (.00033)
34	3.87517	.17651* (.05001)	.0175* (.002)	.0213* (.008)	-0.12524	-.00149* (.00011)	-.00181* (.00019)
35	3.56879	.13546* (.03512)	.0198* (.0034)	.0229* (.0045)	-0.13236	-.00059* (.0002)	-.00068* (.00029)
36	2.91636	.14470* (.05188)	.0247* (.0032)	.0289* (.0049)	-0.18518	-.00158* (.0002)	-.00185* (.0008)
37	2.71494	.18961* (.04335)	.0134* (.0032)	.0165* (.0044)	-0.7926	-.00113* (.00015)	-.00139* (.00023)
38	2.4206	.30213* (.07168)	.0197* (.0029)	.0282* (.0037)	1.0514	-.0012* (.00019)	-.00172* (.00027)

Note: Number in parentheses are the estimated standard errors of the respective coefficients.

* Denotes estimates significantly different from zero at the 0.05 level.

$$\lambda_K = \frac{\hat{\beta}_2}{1 - \hat{\beta}_1}$$

and u_L , u_K are random errors.

Zellner³⁰ calls the model (4.43) SURE model (Seemingly Unrelated Regression Equations). One way to estimate the model (4.43) is to use the least squares technique for each equation. But the least squares estimator ignores the information across the equations given by $\text{cov}(u_L, u_K)$.

Although it is possible to use OLS to estimate the productivities—

³⁰ This procedure follows from seminal work for grouping equations done by Zellner (1962, 1963), followed by Kakwani (1967) and Kmenta and Gilbert (1968).

(4.43)—not efficient estimators would be obtained as a result of the correlation between the disturbance terms. The benefits of SURE are derived when the disturbances from one regression are correlated with the disturbances from another regression.

In our case the correlation was greater than 0.7 in all nine industries. The important implication of $\text{cov}(u_L, u_K)$ is that productivities are jointly determined.

It should be noted that Zellner's estimation technique for seemingly unrelated regressions produces maximum likelihood estimates.

Coefficient estimates³¹ obtained by the application of Zellner's seemingly unrelated technique to the model described by the equations of (4.43) are presented in Table 8.

The results in this phase of the study were most gratifying. Of the 45 parameter estimates ($\hat{\sigma}$, $\hat{\alpha}_2$, $\hat{\lambda}_L$, $\hat{\beta}_2$ and $\hat{\lambda}_K$) all were significant at the 95 % level.

In analyzing the results of the Table 8, the elasticities of substitution are always significant and between zero and one. In all cases the coefficient λ_L is positive and significant at or above the 95 percent level.

One of the most important conclusions from the estimation of the model (4.43) is the stability of λ_L as well as the elasticity of substitution.

The coefficient λ_K was significant at the 95 % level; however, having negative sign. The results presented here indicate that this negative λ_K is stable through the 1963–1980 trend.

Kalt also found negative λ_K suggested by our results. However, Kalt's results are based primarily on an aggregate time series analysis. But in Kalt's study, λ_K , is insignificantly different from zero. It should be pointed out that according to Kalt's findings the average increase in the efficiency of labour was approximately 2.2 % per annum and the corresponding figure for capital was -0.69% . For us, these figures, from Table 8, are 2.82 % in the case of the industry 38 and -0.172% (λ_K), respectively. The figures for labour efficiencies (λ_L) are strikingly close.

4.6 A Dynamic generalization of the Kalt's model

The system (4.43) examined in the above section assumes instantaneous adjustment of the equilibrium level of productivity of labour (or of productivity of capital) to any change in the price of labour (or of capital); i.e. we have assumed that

³¹ Summary statistics (R^2 and $D.W.$) are not applicable for the seemingly unrelated regression results, so that the contribution of this technique may be best measured by the standard errors of the coefficients.

$$\log \left(\frac{y}{L} \right)_t = \log \left(\frac{y}{L} \right)_t^*$$

$$\log \left(\frac{y}{K} \right)_t = \log \left(\frac{y}{K} \right)_t^*$$

where the symbol “*” denotes the equilibrium value.

Firms adjust their output in response to changes in input prices and/or in response to the demand for its output. This implies that, in any empirical investigation concerning output-input relationships, it is necessary to consider this adjustment process. Kalt, however, did not consider any adjustment rule.

The purpose of this section is to extend the Kalt's model by incorporating an adjustment process. Assuming expectations of $\log(y/L)^*$ and $\log(y/K)^*$, we can write (4.43) as

$$\left. \begin{aligned} \log \left(\frac{y}{L} \right)_t^* &= \alpha_0 + \alpha_1 \log w + \alpha_2 t^2 + u_L \\ \log \left(\frac{y}{K} \right)_t^* &= \beta_0 + \beta_1 \log r + \beta_2 t^2 + u_K \end{aligned} \right\} \quad (4.45)$$

Consider now the standard adjustment equations:

$$\frac{\left(\frac{y}{L} \right)_t}{\left(\frac{y}{L} \right)_{t-1}} = \left[\frac{\left(\frac{y}{L} \right)_t^*}{\left(\frac{y}{L} \right)_{t-1}} \right]^\delta \quad 0 < \delta \leq 1 \quad (4.46)$$

$$\frac{\left(\frac{y}{K} \right)_t}{\left(\frac{y}{K} \right)_{t-1}} = \left[\frac{\left(\frac{y}{K} \right)_t^*}{\left(\frac{y}{K} \right)_{t-1}} \right]^\delta \quad 0 < \delta \leq 1 \quad (4.47)$$

or

$$\log \left(\frac{y}{L} \right)_t^* = \frac{1}{\delta} \log \left(\frac{y}{L} \right)_t - \left(\frac{1-\delta}{\delta} \right) \log \left(\frac{y}{L} \right)_{t-1} \quad (4.48)$$

$$\log \left(\frac{y}{K} \right)_t^* = \frac{1}{\delta} \log \left(\frac{y}{K} \right)_t - \left(\frac{1-\delta}{\delta} \right) \log \left(\frac{y}{K} \right)_{t-1} \quad (4.49)$$

where δ is the percentage of total desired adjustment that is achieved during time period t .

Replacing $\log(y/L)_t^*$ and $\log(y/K)_t^*$ in (4.45) by the right hand side of (4.48) and (4.49) we obtain

$$\left. \begin{aligned} \log \left(\frac{Y}{L} \right)_t &= \alpha_0 + \alpha_1 \log w + \alpha_2 \log \left(\frac{y}{L} \right)_{t-1} + \alpha_3 t^2 + u_L \\ \log \left(\frac{Y}{K} \right)_t &= \beta_0 + \beta_1 \log r + \beta_2 \log \left(\frac{y}{K} \right)_{t-1} + \beta_3 t^2 + u_K \end{aligned} \right\} \quad (4.50)$$

where

$$\delta = 1 - \hat{\alpha}_2, \hat{\alpha}_1 = \hat{\beta}_1, \hat{\alpha}_2 = \hat{\beta}_2$$

$$\hat{\sigma} = \frac{\hat{\alpha}_1}{1 - \hat{\alpha}_2}$$

$$\lambda_L = \frac{\hat{\alpha}_3}{(1 - \hat{\sigma}) \delta}$$

$$\lambda_K = \frac{\hat{\beta}_3}{(1 - \hat{\sigma}) \delta}$$

It may be noted that the model (4.50) contains the restriction that coefficients α_2 and β_2 are equal. This must be the case since the estimated elasticity of substitution from labour productivity equation is the same with that of capital productivity equation.

The results of iterative Zellner efficient procedure to the model (4.50) are presented in Table 9. All the estimates of the elasticity of substitution are statistically significant different from zero and unity. Table 9 is self explanatory in the sense that it shows that the industries are operating with non unitary elasticity of substitution. Overall, the estimates of Bergström and the estimates of Bergström and Melander and our estimates of the elasticity of substitution paint a remarkably consistent picture.

The lagged value of the dependent variable, in each equation (4.50), is found to be significantly different from zero in 7 out of 9 industries. The speed of adjustment is approximately 93 %. In all nine industries the estimated coefficients, λ_L , are positive. A negative estimate of the rate of capital augmentation, λ_K , has been obtained for all industries.

Classification of industries according to the estimates of σ , λ_L and λ_K shows that all nine industries enjoy $0 < \sigma < 1$ and $\lambda_L > 0$, $\lambda_K < 0$.

The hypothesis that the rate of capital augmentation, λ_K , is positive is not supported. The coefficient, λ_K , has negative signs through all nine industries. Thus, the sign of λ_K are negative and similar to the results of the previous model (4.43). Again, the results do not support the $\lambda_K > 0$ hypothesis.

It is interesting to compare the results of Table 8 with those of Table 9.

Model (4.50) presents estimates of the production technology with adjust-

bie 9. *Seemingly unrelated estimates of*

$$\left(\frac{Y}{L}\right)_t = \alpha_0 + \alpha_1 \log w_t + \alpha_2 \log \left(\frac{Y}{L}\right)_{t-1} + \alpha_3 t^2 + u_L$$
$$\left(\frac{Y}{K}\right)_t = \beta_0 + \beta_1 \log r_t + \beta_2 \log \left(\frac{Y}{K}\right)_{t-1} + \beta_3 t^2 + u_K$$

mple period 1963–1980

Estimated coefficients										
C	$\hat{\alpha}_0$	$\hat{\alpha}_1=\hat{\beta}_1$	$\hat{\alpha}_2=\hat{\beta}_2$	$\hat{\sigma}$	$\hat{\alpha}_3$	$\hat{\lambda}_L$	$\hat{\beta}_0$	$\hat{\beta}_3$	$\hat{\lambda}_K$	δ
6.13022	.13617*	.00933	.13745*	.0069*	.0081	.25050	-.00179*	-.00209	.99	
	(.0549)	(.01211)	(.0552)	(.0019)			(.00012)			
4.17684	.28684*	.03173*	.29624*	.0189*	.0277	-.02771	-.00081*	-.00119	.96	
		(.04693)	(.01539)	(.05732)			(.00014)			
3.4471	.2209*	.05174*	.23295*	.0129*	.0177	-.00155*	-.00155	-.00213	.94	
	(.05370)	(.01875)	(.05683)	(.0038)			(.00018)			
2.9355	.16958*	.07842*	.18401*	.0159*	.0211	.40261	-.00069*	-.00092	.92	
	(.0834)	(.02236)	(.08938)	(.0030)			(.00026)			
3.6462	.16581*	.06614*	.17755*	.0145*	.0189	-.1205	-.00137*	-.00178	.93	
	(.0469)	(.02057)	(.04921)	(.0027)			(.00011)			
3.5676	.13265*	.00287	.13303*	.0197*	.0228	-.1374	-.00059*	-.00068	.99	
	(.03453)	(.01379)	(.03786)	(.0034)			(.00021)			
2.4913	.19152*	.10371*	.21368*	.0182*	.0258	-.1226	-.04943	-.07014	.89	
	(.0594)	(.0276)	(.06315)	(.0028)			(.12791)			
2.6873	.16984*	.02963*	.17503*	.0131*	.0164	-.8047	-.00109*	-.00136	.97	
	(.0343)	(.01439)	(.03963)	(.003)			(.00015)			
2.0209	.36728*	.07394*	.39661*	.0144*	.0239	1.1654	-.00105*	-.00174	.92	
	(.08316)	(.02053)	(.09435)	(.0024)			(.00021)			

re: Number in parentheses are the estimated standard errors of the respective coefficients.
denotes estimates significantly different from zero at the 0.05 level.

ment process. In model (4.43) we assume instantaneous adjustment. That is, the dynamic model (4.50) has been restricted so that the coefficients $\alpha_2=\beta_2=0$. In model (4.50) out of the nine α_2 parameter estimates, seven are statistically different from zero at the 95 percent level of confidence. These significant coefficients indicate that reduction to a static specification (4.43) would be inappropriate.

We proceed further to test the validity of the restriction $\alpha_2=\beta_2=0$ imposed on model (4.50). We employ test statistics based on the likelihood ratio, defined to be the ratio of the maximum value of the restricted likelihood function (L_R) to the maximum value of the unrestricted likelihood function (L_u). Under the null hypothesis, minus twice the logarithm of the likelihood ratio is asymptotically distributed as chi-squared (χ^2) with

Table 10. *Test statistics for adjustment restrictions on model (4.50)* χ^2 -Test Results⁽¹⁾

ISIC	$-2(\log L_R - \log L_u)^*$	Choice
31	2.6	Static model (4.43)
32	8.2	Dynamic model (4.50)
341	6.4	Dynamic model (4.50)
342	9.2	Dynamic model (4.50)
34	8.6	Dynamic model (4.50)
35	2.1	Static model (4.43)
36	7.4	Dynamic model (4.50)
37	4.5	Dynamic model (4.50)
38	12.2	Dynamic model (4.50)

Note: (1): Test Results based on models (4.43) and (4.50).

(*) $\log L_u$ is the log of the calculated maximum likelihood function without restrictions and $\log L_R$ with restrictions imposed. The critical value of chi square statistic at 5% significance level is 3.84.

number of degrees of freedom equal to the number of restrictions to be tested.

The test was made for nine industries and in Table 10 we give the test statistic; the calculated chi-squared values exceed the 95 percent critical value in seven out of nine. These results suggest that in seven cases the estimated model (4.50) was accepted. We conclude that the specification (4.43) is inappropriate in explaining Swedish technology in the industries 341, 342, 34, 35, 36, 37 and 38.

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Conclusions

Theoretical and empirical conclusions are summarized in this chapter.

Theoretical conclusions

1. The concept of almost homogeneity has played an important role in the analysis of this thesis. The analysis of Chapter 2 began by considering a definition of almost homogeneity, and continued with an application of this to production theory of the firm.

Having established the existence of the almost homogeneous production function, our next objective was to specify a particular algebraic form of the almost homogeneous production function. Consequently, attention was next shifted to the solution of the modified Euler equation and a particular solution yielded the almost homogeneous function $y = (ax_1^{1/k_1} + bx_2^{1/k_2})^k$ in a two input spaces without making any assumption regarding the market conditions. The production function of this form has been used empirically by Dhrymes and Kurz without stating any theoretical argument about the derivation of such a functional form and without pointing out its theoretical generality. Obviously, the solution can be extended in three input spaces and the resulting production function is of the well known Mukerji type.

2. The virtue of our analysis so far has been its generality. It has indicated that using the almost homogeneity property, a nonhomogeneous function can be derived. We then explore the neoclassical properties of the almost homogeneous production function.

We found that the elasticity of substitution of almost homogeneous function of the Dhrymes-Kurz¹ type is constant along the curve $x_2^{1/k_2} = \gamma x_1^{1/k_1}$, but variable along an isoquant.

We show that the almost homogeneous production function of the Dhrymes-Kurz type is neither homogeneous nor homothetic. Consequently, we have proved that this production function is pseudo-homothetic.

¹ See Chapter 2 equation (2.16).

However, the Dhrymes-Kurz production function excludes the possibility of non-almost homogeneous production functions.

This study is the first to examine the Zellner-Revankar differential equation in the class of almost homogeneous functions. Following Zellner and Revankar the almost homogeneous production function was generalized to include variable returns to scale.

It is also the first to utilize the Box-Cox transformations in studying non-almost homogeneous production functions.

Within Chapter 2 we have extended the almost homogeneous production function to a non-homogeneous production function of the Box-Cox type which contains the Dhrymes-Kurz production function as a special case. We have also proved that the specification of the Box-Cox type non-almost homogeneous production function is attractive because it allows returns to scale to be explained by the constant term.

3. In recent years, various algebraic forms emphasizing different aspects of production have been proposed. However, little attention has been given to the specification and the role of the stochastic error term in the empirical production analysis.

The Zellner, Kmenta and Dreze (ZKD) approach, see Chapter 3, to empirical production analysis has provided a much better understanding of the different types of stochastic error structures. Their formulations starts by asserting that the appropriate objective of a firm facing an uncertain production outcome is to maximize expected profits. This objective function, when applied to the Cobb-Douglas or the CES function, has the property that the reduced forms of the inputs are independent of the stochastic error term.

Therefore, the ZKD methodology allows for consistent estimation of the production function parameters by direct application of single-equation methods.

Research analyzing the robustness of the ZKD methodology has been carried out by Blair and Lasky, Grossman and by Feldstein. One of the major aims of Chapter 3 is to simplify Feldstein's model. The Feldstein model was extended by incorporating the assumption that the exponents of the Cobb-Douglas production function are random variables. Employing the same methodology as Feldstein, we have shown that the ZKD result holds when the exponents are random variables.

Empirical conclusions

Three major conclusions can be drawn from the empirical work of this thesis.

1. The way the labour input of the production function is defined and measured is crucial for the applicability of an empirically estimated production function. In most empirical studies of production functions, the labour input has generally been assumed to be homogeneous.

In Chapter 3, we have attempted to throw some light on the behaviour of the aggregate labour function in the case of Swedish manufacturing.

The contribution of this chapter is to provide a simple econometric model, based on the theory of almost homogeneous functions, and show that the gains from using such an aggregation may be large. The advantage of the model employed is that it includes as special cases a number of other models. It may also be noted that unlike most labour aggregation functions currently in use, the almost homogeneous form can be estimated without imposing homogeneity or substitutability restrictions, and it is this feature that make the almost homogeneous function especially attractive.

From the entire class of almost homogeneous functions we have specified the most simple to be applicable to empirical analysis. In the case of Swedish manufacturing industries we have employed the almost homogeneous function which possesses the property that the CES functional specification is a special case. The parameter of the almost homogeneous labour model have been estimated in order to obtain the aggregate labour input. The econometric results using Swedish manufacturing data support the use of this new function as the way of aggregating the heterogeneous components of labour input in two cases. Therefore, the empirical results for these industries rejects the direct aggregation of production workers and non production workers.

Also, our results tend to support the existence of a CES labour input in some industries within the manufacturing sector. These industries are 32 and 34. In two industries—31 and 341—our empirical result supports Tinbergen's hypothesis, i.e., reveals the existence of a Cobb-Douglas labour aggregation. The empirical results also support the existence of a linear labour input in three cases (35, 37 and 38).

In light of these empirical results, it is perhaps not surprising to come to the conclusion that the labour input specification varies from industry to industry.

In this empirical analysis of labour input specification attempts have been made to estimate the aggregate labour input. This method suffer from a number of shortcomings. Among the most important of these is that no account is taken of the existence of different educational levels of labour, the changing educational attainments, the new young entrants to the labour force nor of demographic factors.

2. The production function employed in Chapter 3 was of the Cobb-Douglas type. This was used mainly because of its ease in manipulation and interpretation.

(i) Two different hypotheses are tested concerning the specification of the stochastic error term, and the required statistical technique for testing the relevant models is presented.

The ordinary least squares and non-linear estimation procedures were used. The econometric models yielded information on the characteristic of production. The results indicate that the expected profit maximizing model was accepted in each industry.

(ii) The test for constant returns to scale is accepted for all nine industries.

3. Because the elasticity of substitution between capital and labour in the case of the Cobb-Douglas production function is assumed to equal unity and because we cannot distinguish between neutral and non-neutral technological progress, the two-stage CES production function has been used in Chapter 4. Thus one is able to estimate the elasticity of substitution ($=\sigma$) and the bias of technological progress.

The conclusions reached in Chapter 4 can be summarized as follows:

(i) the graphic method, Resek's test, was first used to examine the nature of technological progress. Our graphs show that the assumption of neutral technological progress may not be valid;

(ii) the evidence of this chapter strongly suggest that the simple time trend is not an appropriate specification for the technological progress in the case of Swedish manufacturing;

(iii) it was found that the estimates of the elasticity of substitution are sensitive to the specification of technological progress;

(iv) the elasticity of substitution between capital and labour appears to be between zero and one for all nine industries. Thus, the CES production function is more appropriate than the Cobb-Douglas production function for the Swedish manufacturing sector;

(iv) using the dummy variable approach we found that the elasticity of substitution has not changed from period to period;

(v) the method of the aggregating labour components affects the of elasticity of substitution between capital and labour. The CES production function in which the labour input is specified as an almost homogeneous function gave lower elasticities of substitution and a better statistical fit than a CES with a linear aggregation of the labour input;

(vi) alternative estimations of σ with a different technique (2SLS) for the purpose of discovering the sensitivity of σ and the simultaneity bias showed that variations in statistical techniques does not significant inference on

these estimates. The results of 2SLS provide further support that $0 < \sigma < 1$. Therefore, the OLS and 2SLS estimates are remarkably consistent with the time-series estimates of the elasticity of substitution ($0 < \sigma < 1$). Furthermore, 2SLS results are qualitatively similar to the OLS estimates;

(vii) since in all nine industries we have found that $\sigma < 1$, and technological progress increased the efficiency of labour faster than that of capital, technological progress has been biased in a capital using direction over the period 1963–1980. It is remarkable to observe that in no industry was labour-using found. This means that there was a labour-augmenting bias in technological progress in the Swedish manufacturing industries during the period in question.

Whilst direct comparison with Bergström's estimates is not strictly valid because of different data classification and estimation techniques, it would appear that the present results agree with his findings that: "Labour augmenting technological progress seems to predominate over capital augmenting growth which, combined with our information about the elasticity of substitution, means that technological development has been labour-saving."

The picture that emerges from the results of Chapter 4, as a whole, agrees with the general findings of empirical studies using different levels of aggregation and a different theoretical framework. We found that technological progress has been labour augmenting.

This result agrees with the findings of professor Bentzel:² "the calculations strongly indicate that technological progress was predominantly labour-augmenting. It is certainly true that the simulations are based on an assumption that this progress was labour-augmenting and that therefore they cannot tell us anything about the possible existence of other types of progress. However, the assumption made in this respect has not come forth by change. In fact a great number of experiments have been performed and all of them have given unique results; namely if other types of technological progress are introduced instead of the labour-augmenting factor the model cannot mirror the actual values at all, and if other types of technological progress are introduced as supplements to a labour-augmenting factor the empirically visible effects of this progress will be small and erratic."

The productivities of labour and capital have not been extensively used simultaneously in studying the rates of augmentation and the elasticity of substitution. The technique used to simultaneously estimate the elasticity of substitution and the rate of change of the efficiency of labour and capital is originally due to Kalt and Bergström-Melanders.

² See Bentzel (1978).

Table 11. *Elasticities of substitution of Swedish manufacturing industries. Sample period 1963–1980*

ISIC	Estimated elasticities of substitution			
	Cost-minimizing Model Table 4	Cost-minimizing Model (2SLS) Table 7	Classical seemingly Unrelated model Table 8	Dynamic seemingly Unrelated model Table 9
31	.16997 (.08452)	.18807 (.08468)	.19058 (.04061)	.13745 (.0552)
32	.23366 (.09719)	.27886 (.12989)	.27208 (.06512)	.29624 (.05732)
341	.15449 (.06332)	.24857 (.11958)	.22453 (.05876)	.23295 (.05683)
342	.13499 (.05741)	.21309 (.10963)	.20357 (.10149)	.18401 (.08938)
34	.13920 (.0572)	.17135 (.08241)	.17651 (.05001)	.17755 (.04921)
35	.20313 (.05697)	.41103 (.12429)	.13546 (.03512)	.13303 (.03786)
36	.35179 (.12217)	.29231 (.13256)	.14470 (.05188)	.21368 (.06315)
37	.16332 (.06108)	.37989 (.15299)	.18961 (.04335)	.17503 (.03963)
38	.37657 (.15856)	.38639 (.16552)	.30213 (.07168)	.39661 (.09435)

Note: Number in parentheses are the estimated standard errors of the respective coefficients.

Some econometricians have tackled the estimation of this type of model through the SURE technique. The empirical results are satisfactory. It is interesting to note that:

(a) The elasticity of substitution between capital and labour appear to be between zero and one for all nine individual industries. Thus the CES production function is more appropriate than the Cobb-Douglas production function for the Swedish manufacturing sector.

(b) It is interesting and useful to compare the four sets of estimates of the elasticity of substitution presented in this thesis.

Table 11 presents four estimators of the elasticity of substitution. It is clear from these empirical results that they are sensitive to the method chosen for estimating the elasticity of substitution.

The first column contains the ordinary least squares (OLS) estimators under the cost minimization assumption.

The second column presents estimators of the elasticity of substitution

which are based upon cost minimization but were estimated using two-stage least squares (2SLS).

The third and fourth column contains the estimators of the elasticity of substitution but they were estimated using Zellner's seemingly unrelated technique (SURE).

All the elasticities of substitution estimators based on 2SLS are greater than those estimated by OLS. The standard errors in the OLS estimation are in all cases smaller than their counterparts arising from 2SLS.

The results of both the OLS and 2SLS techniques suggest that the elasticity of substitution is less than one; these results are also supported by the SURE (columns 3 and 4) results.

It is interesting to note that the use of the seemingly unrelated regressions technique results in lower standard errors. The higher the correlation between residuals, the larger the gain by the Zellner's method. In all cases the correlation was greater than 0.7. Therefore, Zellner's seemingly unrelated technique performs best, i.e., exhibits the smallest standard errors (columns 3 and 4). Pairwise, comparisons between all pairs of estimates using the standard errors gave a ranking which puts SURE on top followed by OLS and 2SLS. Therefore, judging by the relative efficiency criterion, the SURE demonstrated its minimum-variance property in a striking fashion, the OLS is second best and 2SLS is third best. In other words, it appears to be more advantageous to use SURE rather than OLS and 2SLS.

The estimates of λ_L are positive while the estimates of λ_K appear to have been negative and significantly different from zero in all nine industries.

The negative and significant value of λ_K open a range of questions that surely will be of concern to the researchers of industrial growth in the Swedish manufacturing sector.

How can we explain the negative of capital augmentation together with a positive of labour augmentation?

Samuelson, in his paper "Rejoinder: Agreements, Disagreements, Doubts and the Case of Induced Harrod-Neutral Technical Change"³ recognized the possibility of negative factor augmentation. According to Samuelson "one factor augmentation can go negative if the invention makes the other factor augmentation sufficiently positive". This argument is represented by the innovation possibility frontier.

This type of curve was introduced by Kennedy.⁴ Kennedy showed that there exists a trade-off between capital augmentation and labour augmentation; the greater is the reduction in labour requirement, for example, the smaller is the reduction in capital requirements.

³ See Samuelson (1965 and 1966).

⁴ See Kennedy (1964, 1966 and 1967). For detailed discussion of Kennedy's model, see Drandakis and Phelps (1966).

Kennedy asserted that, after a certain point, a further increase in the rate of labour (capital) augmentation is necessarily accompanied by a negative rate of capital (labour) augmentation. Therefore, Kennedy's theory of induced innovation was useful in explaining the negative capital augmentation in Swedish manufacturing.

Thus, the sign pattern ($\lambda_L > 0$ and $\lambda_K < 0$) is not inconsistent within the context of Kennedy's model.

Kennedy's analysis has a number of obvious weaknesses which we shall mention briefly.⁵

First, Kennedy's theory of induced innovation does not completely solve the problem of the direction of technological bias. Second, Kennedy's theory does not provide the background for a theory of endogenous bias. In Kennedy's theory, technological progress is represented as an exogenous process, i.e., technological progress is disembodied.

Several models of induced factor bias have been suggested in the literature. The first attempt to develop a microeconomic approach to induced innovation was made by Ahmad⁶ in a seminal paper in 1966.

Ahmad employed the concept of an innovative possibility curve (IPC) which is the envelope of "... all alternative isoquants (representing a given output on various production functions) which the businessman expects to develop with the use of the available amount of innovating skill and time".

As factor input prices change over time, the firm selects a new production isoquant consistent with cost minimization. The end result is induced factor-input bias. In Ahmad's analysis, technological progress will affect the factors of production that have been introduced after the date of innovation. This is likely to take place with embodied technological progress.

In our case, we have found that the bias of technological progress in Swedish manufacturing is not neutral. In fact during the period 1963–1980 the rate of increase in real wage rate, $\dot{w}/w$, is greater than the price of capital.

Our empirical results strongly support the view that Swedish industries sought capital-using techniques in response to expensive labour. Since the wage rate increased in Swedish manufacturing during the period 1963–1980, we can say that entrepreneurs were induced to save labour. Therefore, one can say that factor prices were essential in inducing the entrepreneurs to save on the factor whose price has increased.

⁵ The assumptions which the Kennedy growth model approach to induced innovation carries have been examined by Nordhaus (1967, 1973), Wan (1971), and in Binswanger and Ruttan (1978).

⁶ For a discussion, see Ahmad (1966, 1967 and 1967*a*). Recently Binswanger (1974) has reformulated the microeconomic approach and it was shown that in this reformulation the Ahmad and Kennedy models are special cases of the more general model.

Finally, we should note that our results support Ahmad's hypothesis that factor prices determine the character of factor saving to a great extent. This also implies that technological progress in Swedish manufacturing was embodied during the period 1963–1980. These empirical results are consistent with the general view that technological progress is endogenous to the economic system and that movements in relative factor prices act as a spur to innovation in the direction of saving on the more expensive factor.

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Graphs



Fig. 4

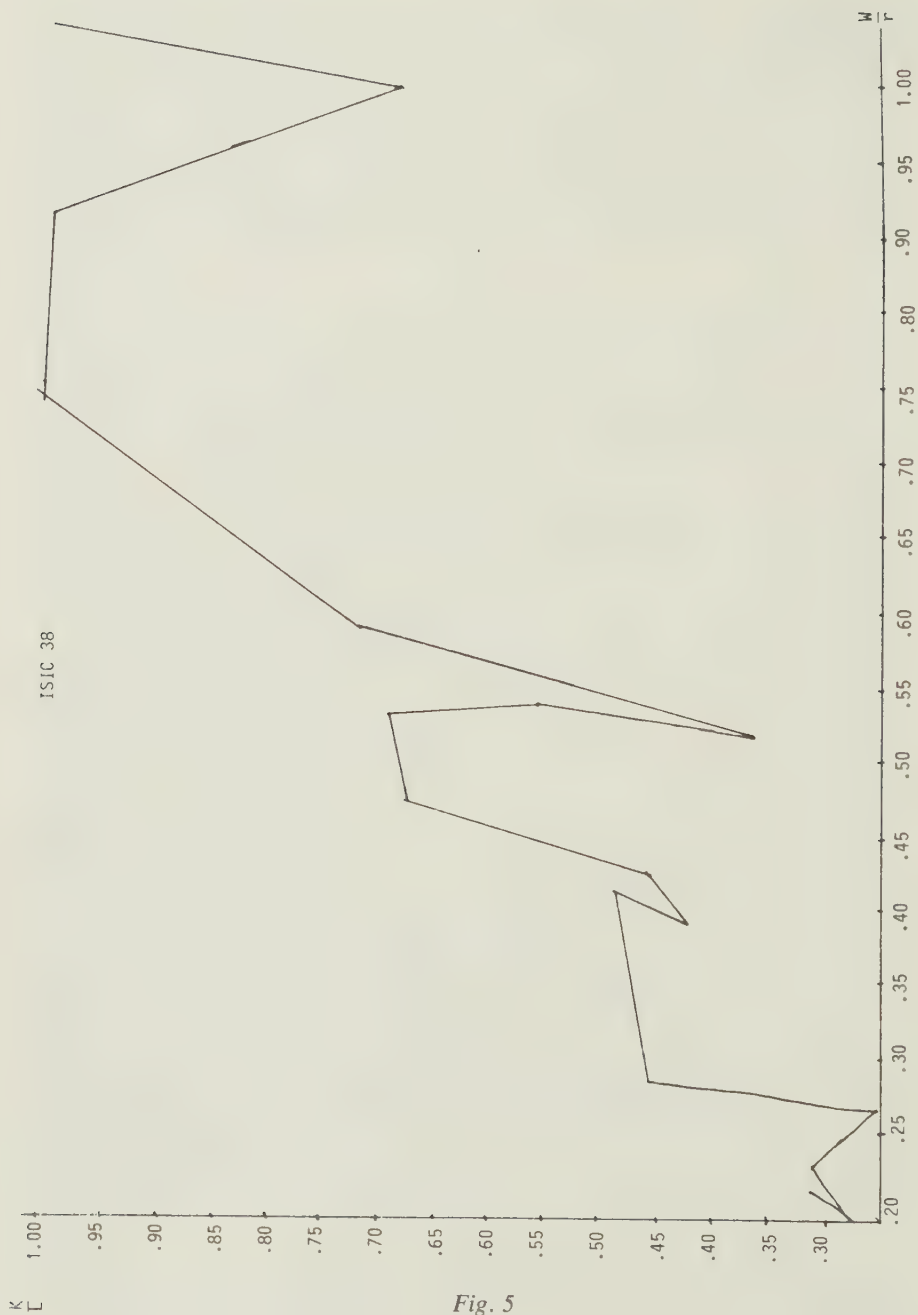


Fig. 5

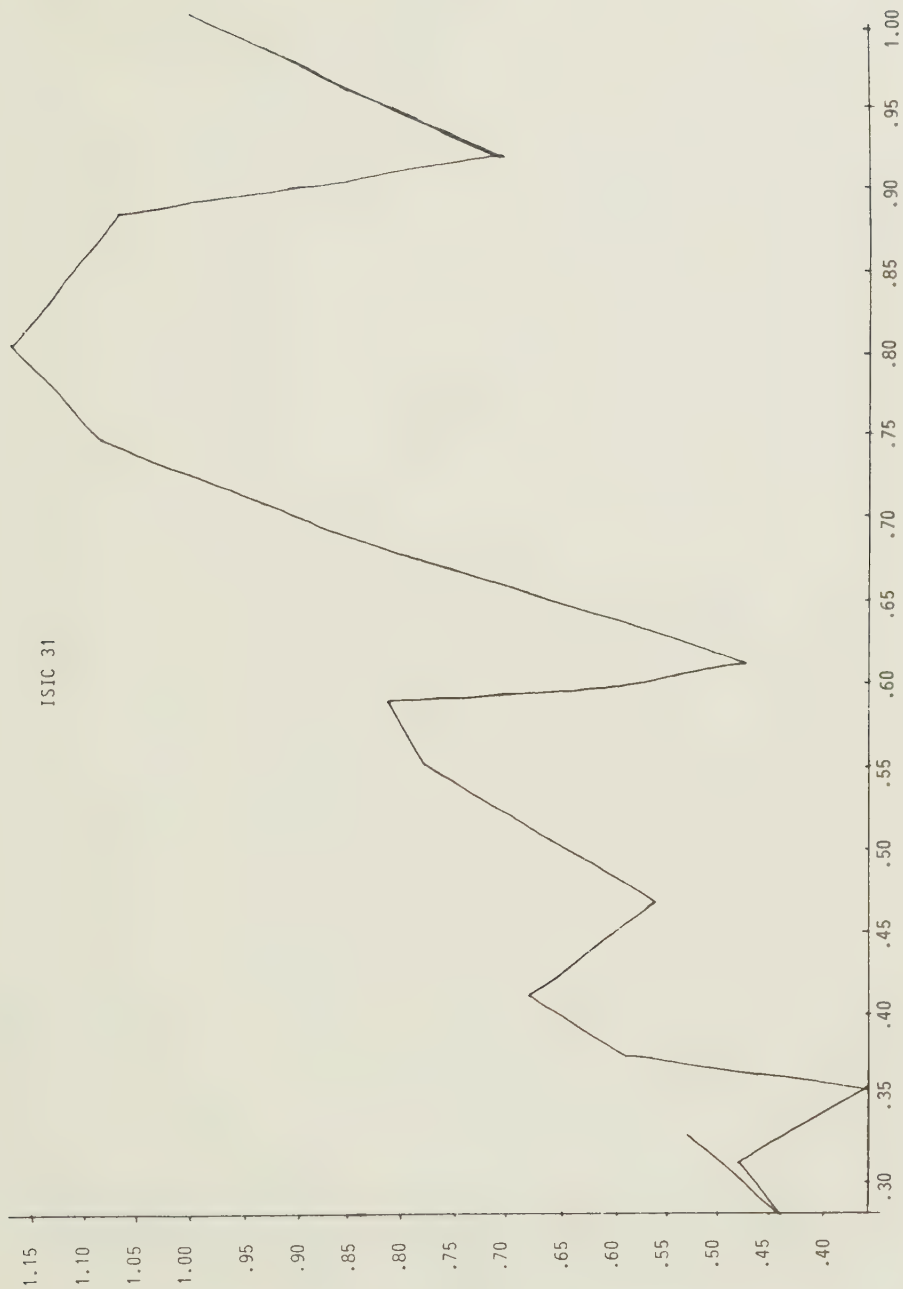


Fig. 6

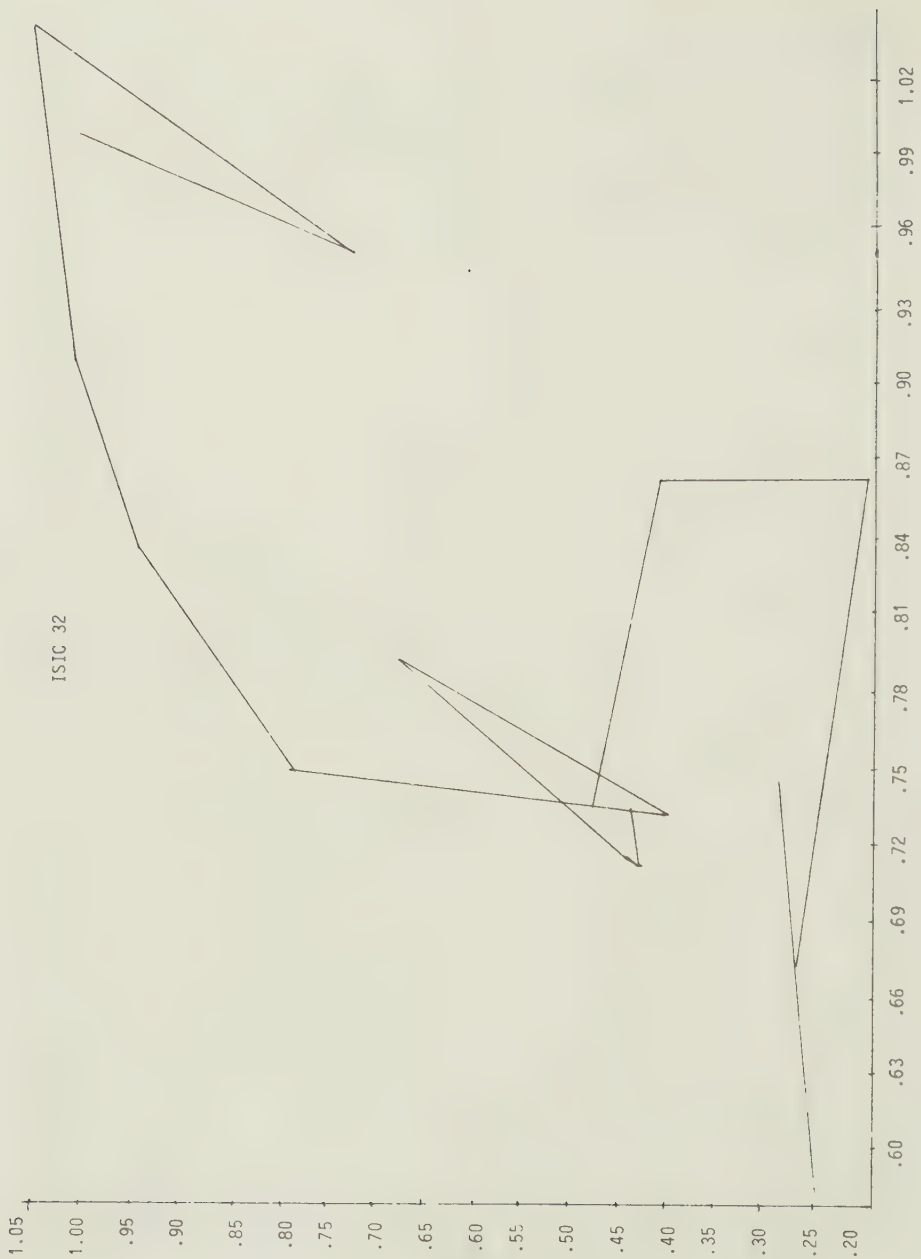


Fig. 7



Fig. 8

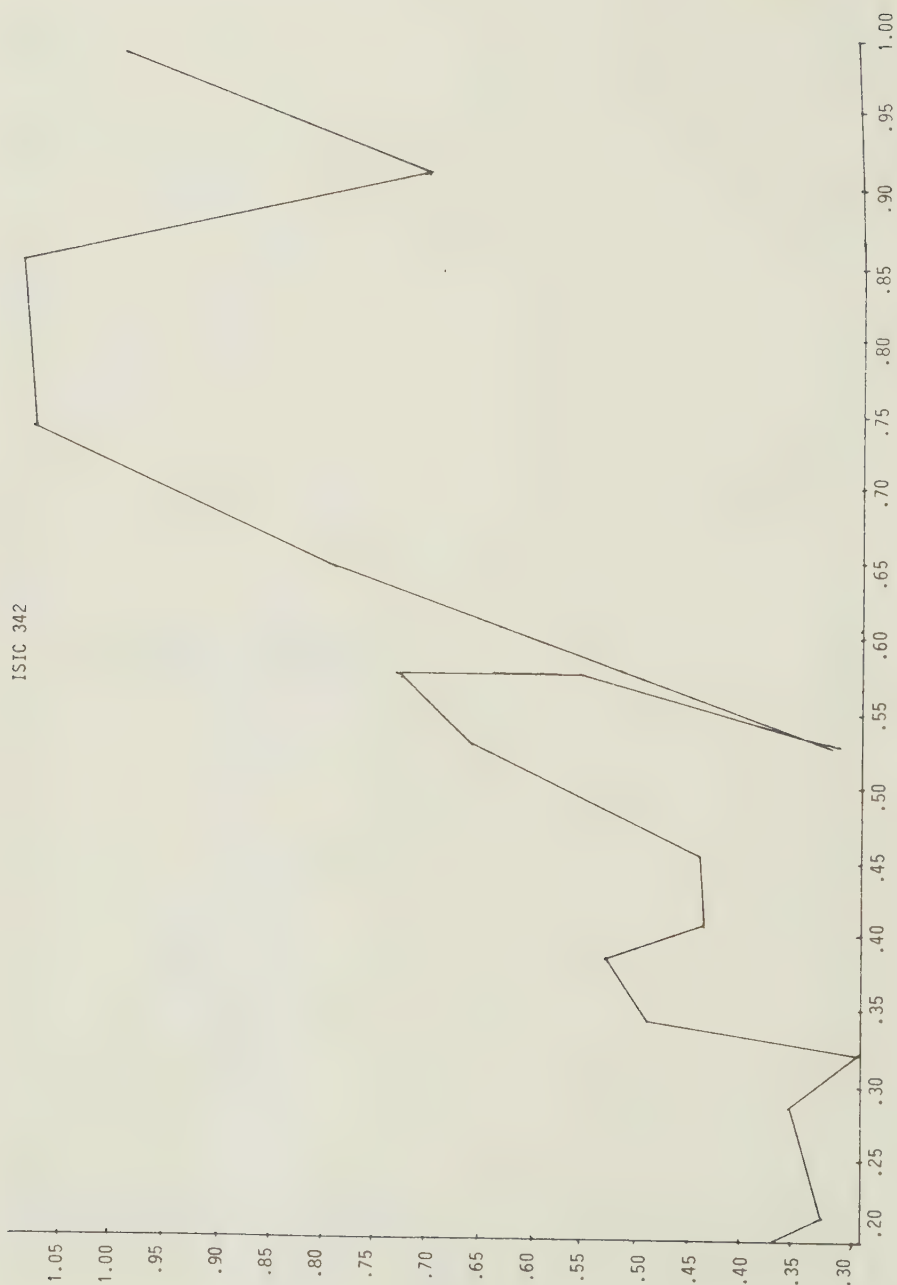


Fig. 9

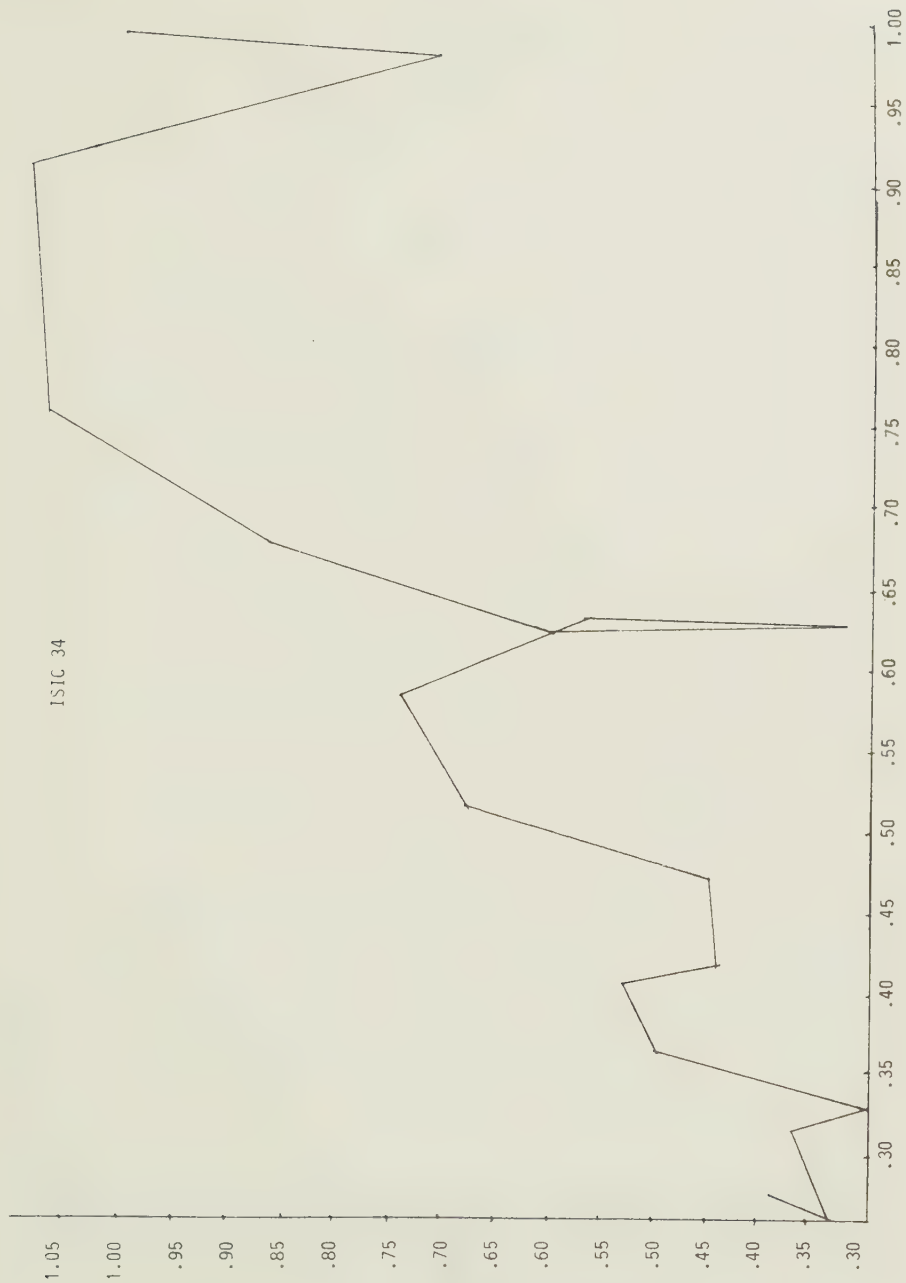


Fig. 10



Fig. 11

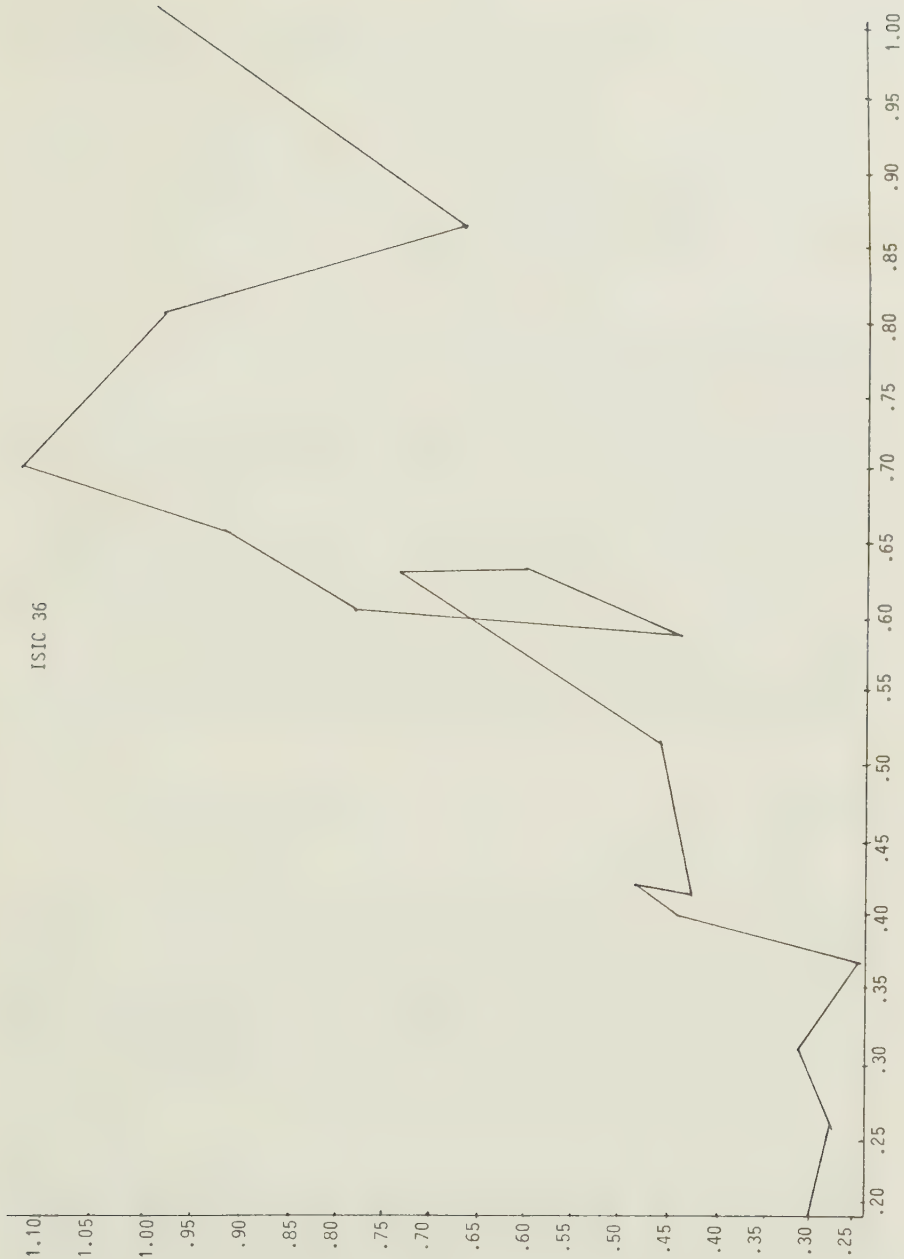


Fig. 12

Data

The variables used in this study are described below:

(i) *Output (y)*: The value added in constant (1975) prices is used to represent output. The basic data on value added at current prices (yy) for the industrial sectors, was obtained from different issues of the annual survey of industries (Data fördelade enligt Standard för svensk näringsgrensfördelning). The deflator P_y for value added was found by dividing the value added at current prices by the value added at 1975 constant prices from National Accounts. Therefore, the real value added for each industry was found by dividing the yy by P_y , i.e. $y=yy/P_y$.

(ii) *Labour Input*: As is customary in the neoclassical production function, labour input will be measured in manhours. For each industry we have:

WE = wage-earners

SE = salaried employees

The various issues of Statistisk Årsbok provides data on hours worked per week, then the hours worked per year can be approximated by multiplying WE by hours worked per week 50 weeks per year.

(iii) *Capital Input (K)*: The conceptual problems associated with the measurement of capital input have been outlined by Griliches and Jorgenson in their paper "Sources of Measured Productivity Change: Capital Input".¹ Griliches and Jorgenson suggest

The measurement of capital services is less straight forward than the measurement of labour services because the consumer of a capital service is usually also the supplier of the service; the whole transaction is recorded only in the internal accounts of individual economic units. The obstacles to extracting this information for purposes of economic accounting are almost insuperable; the information must be obtained by a relatively lengthy chain of indirect inference. First, the calculation begins with data on the values of transactions in new investment goods. These values must be separated into prices and quantities of investment goods. Second,

¹ See Griliches and Jorgenson (1966).

the quantity of new investment goods reduced by the quantity of old investment goods replaced must be added to accumulated stocks. Third, the quantity of capital services corresponding to each stock must be calculated."

However, the problems are more than conceptual. For example, an estimate of capital input requires the appropriate technique for aggregation of different capital components; and in this case there is a disagreement about the appropriate technique, see the debate between Denison, Jorgenson and Griliches, which is appeared in "The Measurement of Productivity".²

The choice of net or gross capital stock is another issue in the measurement of capital. An additional issue is whether to include land and inventories in the measurement of the capital stock. The measurement of the changes in the quality of capital is another issue. Because the capital quality changes are difficult to measure, we do not have direct measures of capital quality. We can, however, observe one factor which should be an important source of the quality of capital: research and development (R and D) expenditures. Here again, we have the problem of considering R and D as a separate factor of production or we must consider R and D expenditures as a component of the aggregate capital input.

Finally, an additional issue is which index of capacity utilization to use. The more serious problem with capacity index appears to be that of defining and measuring. There is no single generally accepted concept of capacity.

The methodologies employed to construct capacity index are undertaken and the index will no longer provide the correct signals. For example, the experience for formulating a capacity index in U.S. indicates that:

... the Federal Reserve index estimates that as of mid-October 1973, a fair amount of spare capacity existed in the American economy. The estimated operating rate was only 83.4 percent for 1973: 3. By contrast, the Wharton index for manufacturing was as high as 96.7 in the same period, and was rising faster than the Federal Reserve index. The McGraw-Hill index was at an intermediate level of 86.5 percent in September. Similar divergences among the indexes had been apparent for several months. *These messages are so different that they suggest the need for a close look into the whole subject* (emphasis added). L. Klein and Long.³

In this study the capital stock for machinery at 1975 prices given by Capital Formation and Stocks of Fixed Capital, (Statistiska Meddelanden N 1981:2.5, Statistiska Centralbyrån, Nov. 1981) have been used. We have not attempted here to multiply the capital stock by capacity utilization index. Since it is extremely hard to estimate the capacity utilization and the measure must be correctly specified we have omitted the adjustment in the

² See Jorgenson (1972).

³ See Klein and Long (1973).

capital input. In this practice, we do not differ from other production analysts.⁴

(iv) Price of labour is computed as the ratio of labour compensation to the total manhours.

(v) The rental price of capital services was computed by Bergström-Södersten.⁵

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⁴ See Denison (1969), and Gollop and Jorgenson (1980) and Kendrick (1976 and 1980).

⁵ See Bergström and Södersten (1984).

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